Multimodal Transport Networks*

Simon Fuchs — Atlanta Fed

Woan Foong Wong — University of Oregon

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Abstract

The movement of goods from origin to destination takes place over multiple modes of transportation. Correspondingly, intermodal terminals play an important role in facilitating transportation over the multimodal network. This paper studies multimodal transport networks and their impact on infrastructure investments. We propose a tractable theory of transportation across domestic transportation networks with multiple modes of transportation by embedding multimodal routes into a spatial equilibrium model with endogenous stochastic route choice. We calibrate the model to US domestic freight flows using high resolution geographic information system (GIS) information and detailed data on traffic along road, rail, and international ports. We estimate the strength of intermodal port congestion from ship dwell times and its multimodal impact on railcar dwell times. We then employ the model to evaluate the welfare effects of terminal investments across the US. We identify important bottlenecks in the US transportation system, with the reduction of the transportation cost by 1 percent in the most important nodes generating welfare gains equivalent to 200-300 million USD of additional GDP (in 2012 USD).

JEL Code: F11, R12, R42

Keywords: Infrastructure investments, Multimodal, Spatial equilibrium

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1 Introduction

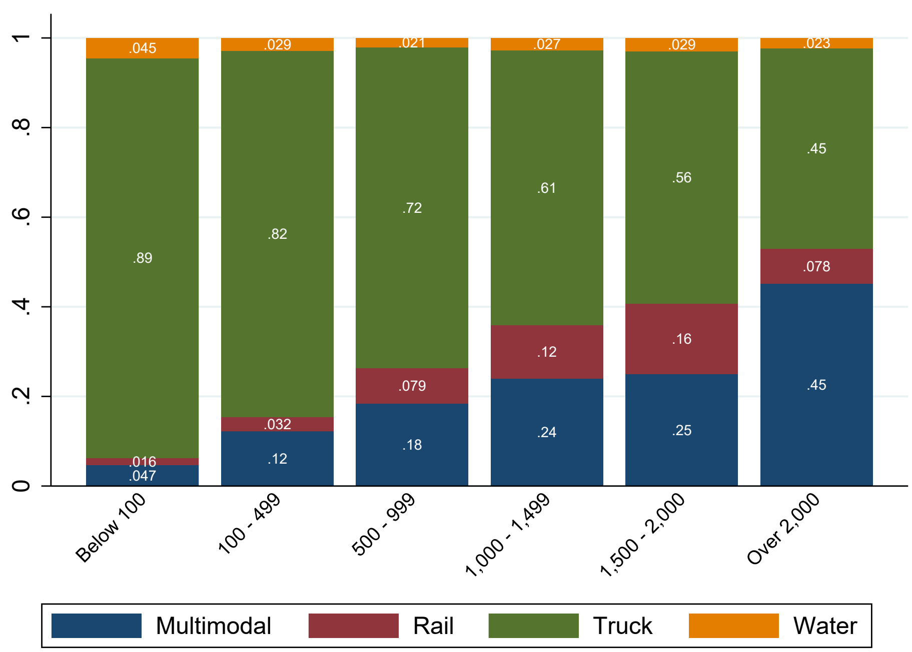
The movement of goods takes place over multiple modes of transportation, including highways, railroads, oceans, and waterways. While trucks are mostly used to move US freight over shorter distances, freight share by railroads and multiple modes increases steadily over longer distances. For freight moved over longer distances of 1000 miles or more, rail and multiple modes of transport accounts for one-third of freight by value (Figure 1) and more than half by weight (Figure A.1). For freight moved over more than 2000 miles, more than half of freight value is transported via rail and multiple modes. For context, the road distance between Los Angeles and Chicago is roughly 2000 miles.

Correspondingly, intermodal terminals play an important role in facilitating how goods are transported over this network. The economic returns from new technologies and infrastructure investments in transportation can depend importantly on the level of integration across modes as well as the presence of bottlenecks at intermodal terminals.

This project studies multimodal transport networks and their impact on the economic and environmental returns to new technology and infrastructure investments. In particular, we focus on how these outcomes will depend on the geography of the multimodal transportation network, the placement of intermodal terminals that allow for switches between modes of transportation, as well as the relative cost of transportation across modes. By incorporating these features we provide a framework that allows us to realistically evaluate infrastructure policies taking the complete domestic transportation network into account. In future work, this also allows us for the first time to evaluate the environmental impact of infrastructure investments that stems from modal substitution.

We first develop a quantitative spatial equilibrium model that incorporates transportation across multiple transport modes. We provide a tractable way of describing the freight forwarding problem in a setting where multiple modes of transportation are available and where modal switch is not restricted except for the geography of the modal transportation network and incurred switching costs. Extending the routing-based formulation of transport cost in Allen and Arkolakis (2022) to allow for multi- and intermodal routing we show how to use the properties of partitioned matrices to derive closed-form expressions for the expected transport cost despite the increased dimensionality and complexity of the underlying transport network. The transportation choice is then embedded into an otherwise standard economic geography model with a constant elasticity import demand system over origin-differentiated goods with a multimodal

Figure 1. US Transport Mode Value Shares by Distance, 2018



Notes: This figure plots the observed value share of cargo transported by different modes across various distances. Multimodal indicates cargo movement that involves more than one mode. Source: Freight Analysis Framework, US Department of Transportation, and authors' calculations.

extension of the routing formulation of transport cost in Allen and Arkolakis (2022), in the following referred to as AA2022. Crucially, we derive a simple set of equations that allows for counterfactual experiments, and in particular allows us to evaluate the welfare consequences of modal or terminal infrastructure improvements.

We motivate the model empirically in two ways. First we examine the impact of congestion at intermodal terminals, both at the terminal and its spillover impact on the multimodal network. Using vessel positioning data down to the minute interval, we estimate a elasticity of port congestion by investigating ship dwell times and their responsiveness to port traffic. We then study the impact of port traffic on the multimodal transport network by focusing on railcar dwell times at local rail stations. Second, we study the double-stacking of intermodal containers in railroad transportation. A unique data set, the Confidential Carload Waybill dataset maintained by the Surface Transportation Board (STB), allows us to trace out the transport flows across the US railroad infrastructure. Using this data we examine a large-scale infrastructure improvement, the Heartland corridor, which improved railroad connections along the Virginia-Chicago corridor in the Northeastern part of the US, and crucially, upgraded the infrastructure to make double-stacking feasible. We examine to what extent this infrastructure measure diverted transport flows towards the railroad system and provide suggestive evidence on diverted flows away from road.

Next, we calibrate the model to fit and reproduce salient features of the US domestic transportation network. We first build a graph representation of the US multimodal transportation system drawing on high-resolution GIS data on road, rail and maritime linkages, as well as the location of intermodal switching facilities. In combination with detailed road traffic and railroad data the model can then be applied to evaluate infrastructure investments, taking the multimodal nature of the US domestic transport system into account. We employ the model to evaluate and compare the welfare impact of investing in different terminals across the country, thus improving the intermodal integration of the primary and secondary transportation network. The analysis points towards substantial and highly heterogeneous welfare gains across space. Investments that would lower transportation costs in the most important nodes by only 1 percent would generate an aggregate welfare gain equivalent to 200-300 million USD of additional GDP (in 2012 USD).

Our paper is related to a number of different strands of research. First, this paper contributes to a rapidly expanding literature incorporating realistic transportation network into quantitative spatial equilibrium models. The quantitative spatial economics literature has developed extremely useful tools that can answer a range of questions taking the underlying spatial distribution of economic activity into account and that can be credibly mapped to dis-aggregated data with a spatial dimension (see Redding (2020) for a recent survey). Within that literature, there have been multiple efforts to merge the dis-aggregated network structure of transportation infrastructure with a general equilibrium economic geography model. (Fajgelbaum and Schaal, 2017; Allen and Arkolakis, 2022). In particular, Allen and Arkolakis (2022) proposed a tractable way of incorporating the optimal routing choice into an spatial equilibrium model, allowing the authors to examine the general equilibrium implications of transportation improvements. While much theoretical progress has been made, the literature has often focused on one mode of transportation, approximating transport costs with either road or maritime transportation costs (Wong, 2020; Ganapati, Wong and Ziv, 2021; Coşar and Demir, 2018; Brancaccio, Kalouptsidi and Papageorgiou, 2020; Heiland et al., 2019). Our project adds to this literature by incorporating multiple modes of transportation as well as intermodal switching terminals and offering a fully-fledged general equilibrium analysis of the US multi-modal transportation system.[1]

[1]Recent exceptions include Fan, Lu and Luo (2019), Fan and Luo (2020), Bonadio (2021) and Jaworski,

Second, our paper is related to a long-standing literature in transportation studies that examines route and mode choice both empirically and theoretically (McFadden, Winston and Boersch-Supan, 1986; Rich, Kveiborg and Hansen, 2011; Beuthe, Jourquin and Urbain, 2014; Winston, 1981). The state-of-the-art in transportation studies solves high-dimensional traffic assignment problems algorithmically accounting for both dis-aggregated heterogeneity in modal and route choice.[2] We employ similar tools to those recently developed in transportation studies. Specifically, we employ what the transportation literature calls a *stochastic user equilibrium* where routes and modes are chosen subject to a stochastic perception error. However, we go beyond the transportation literature, by fulling embedding the stochastic user equilibrium into a spatial general equilibrium framework, where input and output markets across space clear and factor and output prices are endogenously determined.

The remainder of the draft is structured as follows. Section 2 describes the US multimodal transport network and our data. We then detail the multimodal routing model in Section 3 and describe how we calibrate the model to US data in Section 4. We apply our model to evaluate the welfare impact of terminal investments in Section 5 and conclude in Section 6.

2 US Domestic Freight Transportation and Data

In this section, we provide an overview of the US domestic transportation system and introduce our data sources.

2.1 US Domestic Freight Transportation

As mentioned above, the movement of goods from origin to destination takes place over multiple modes of transportation. While trucking is dominant along the shortest distances, alternative modes and multimodal transportation becomes more important over longer distances, as can be seen by the modal split over different distance bands for the US (Figure 1). The geography of the US multimodal transport network includes rail, road, and waterways with intermodal switching

Kitchens and Nigai (2020) also explore the impact of multiple modes on domestic transportation costs. Fan, Lu and Luo (2019) and Jaworski, Kitchens and Nigai (2020) focus on domestic road and highways while Bonadio (2021) focuses on roads and road access to ports. Fan and Luo (2020) is a note which characterizes bilateral transport costs and their elasticities with respect to transshipment costs.

[2] For a recent theoretical contribution compare Kitthamkesorn, Chen and Xu (2015) which solves for the traffic assignment problem allowing for both endogenous route and mode choice. A recent applied quantitative contribution in this literature (Li, Xie and Bao, 2022) models the multimodal linkages between the US and China with endogenous route choice and congestion at port locations.

terminals playing a major role is facilitating the movement of goods between the transport modes (Figure 2). Additionally, the dense road network plays an important role in facilitating transportation at the start and end of the movement of goods. This is commonly known as the first and last mile in freight transportation (Rodrigue, 2020; Ranieri et al., 2018).

Furthermore, the US domestic freight landscape has been changing rapidly. In the 1980s, the top three modes of transporting freight in the United states are truck, railroads, and coast-wise ocean shipping (Figure A.7). However, by 2017, ocean shipping shares have drastically declined while the other two mode shares have increased. Trucking and rail both have been increasing their overall shares by about 10 percentage points by 2017. In 1980, trucking and rail accounted for 41 percent and 30 percent respectively. By 2017, trucking shares have increased to almost 50 percent while rail has increased to about 40 percent. On the other hand, ocean shipping has been declining. Ocean shipping accounted for about 20 percent of US freight in 1980 but this has declined to 4.2 percent by 2017. Inland waterway shipping shares started at 9.3 percent and declined slightly to 7.4 percent. Since air freight shares are small throughout this period (0.15 percent in 1980 to 0.34 percent by 2017), we abstract away from this mode of transport in our analysis.

With rail offering a cost- and energy-efficient alternative to trucking, it is widely expected that rail continues to increase in importance. Since the rail network is not sufficiently dense to directly reach final consumers, nevertheless trucking will remain as one of the only feasible solutions to the last mile problem. This situation emphasizes the importance of understanding the multimodal capacity of the US freight network, the key aim of this paper.

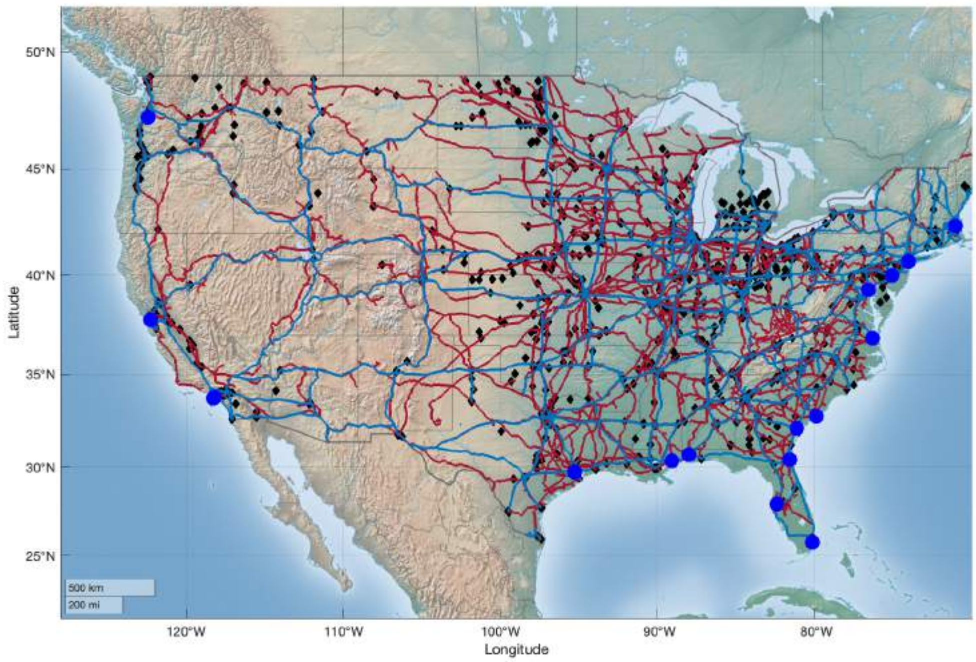
2.2 Data

This subsection introduces the different data sources that we use for our motivating empirical analysis in this section, as well as for our structural analysis in Section 5.

2.2.1 AIS Vessel Traffic Data

We utilize automatic identification system (AIS) vessel traffic data from Marine Cadastre, a joint initiative between the Bureau of Ocean Energy Management and the National Oceanic and Atmospheric Administration. This data captures vessel location in US waters at 1-minute intervals using 200 land-based receiving stations. We observe the vessel's identifying information, its longitude and latitude location down to the minute, speed, and navigation status. The vessel's

Figure 2. US Multimodal Transportation Network



Notes: This figure shows the combined US multimodal freight network. We obtain the original GIS information from the U.S. Census Bureau's Topologically Integrated Geographic Encoding and Referencing (TIGER) Database. The red lines indicate the Class I multimodal railroad network. The blue lines indicate the interstate highway system (IHS). Black diamonds indicate freight terminals that are owned by Class I operators allow for road-to-rail or rail-to-road intermodal movements. The blue circles indicate the top 18 ports.

identifying information includes its International Maritime Organization Vessel number (IMO). The vessel's navigation status captures whether the vessel is being propelled (under way using engine), or moored—held in position at a pier.[3] Using information on the ship's speed and navigation status, we define a ship's dwell time to be the time it spends being moored at a pier and has zero speed. This is a conservative measure of ship dwell time at ports because (1) a ship will spend time navigating within the port area as it prepares to moor at a pier and (2) a ship can also end up waiting outside of the port area at anchor before navigating to the port (New York Times, 2021). In future work we plan on investigating additional measures of dwell times, including the entire time a ship spends within the port areas (not just when they are moored), as well as the time a ship spends at anchor within or just outside of port areas.

In order to match these ships to the ports they are located at, we next require geographical information of the ports. We use the Port Statistical Area shapefiles from the US Army Corps

[3] There are additional AIS navigational statuses than the ones described here, for example being propelled via sail (under way sailing) or at anchor (held in position by an anchor but not at a dock). Future work will consider utilizing additional statuses.

of Engineers and match these ships to the top 30 container ports in the US. These port polygon areas also allows us to calculate the total amount of time a ship spends within the port region on top of the time it spends moored at a dock. Additionally, in order to identify the cargo capacity of these ships and their containership status, we match these ships to the Port Entrance and Clearance dataset from the US Army Corps of Engineers using their identifying information and when they are at these ports. The ship cargo capacity measures the volume of the ship that can be used for loading cargo (also known as net tonnage of a ship). This cargo capacity measure for each ship will contribute to our port traffic measure at each port every day.

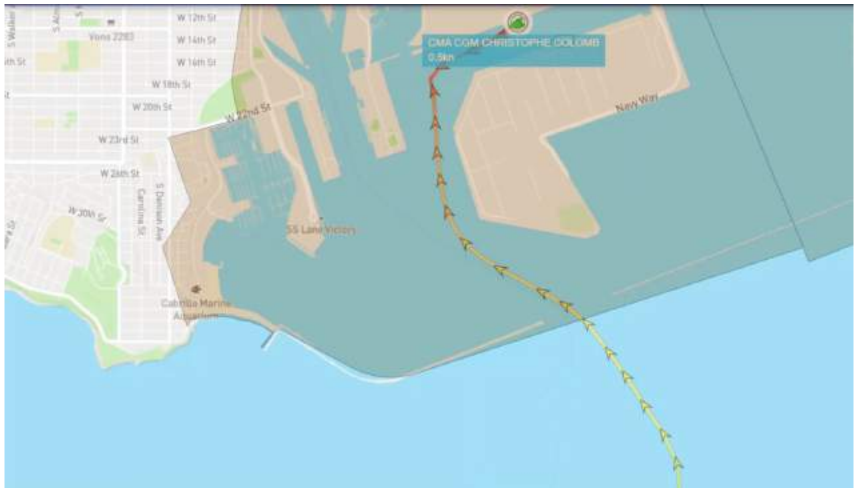
We highlight two examples to show how we capture these ships and the time they spend at a port. Panel (A) Figure 3 shows the path of containership CMA CGM Christophe Colomb as it enters the Port of Los Angeles (LA) on May 2, 2022. It is a containership with a cargo capacity of 86,100 tons (13,800 twenty-foot equivalent unit containers (TEUs)) and is operated by container shipping company CMA CGM. Panel (B) Figure 3 shows the path of containership Guthorm Maersk entering and leaving the Port of Newark. Guthorm Maersk is a containership with a cargo capacity of 57,000 tons (11,000 TEUs) and is operated by container shipping company Maersk. The ship path entering the port is highlighted in the figure and the redder color indicates slower speed. The darker region of both figures indicate the port polygon for both ports as defined by the US Army Corps of Engineers.

Port Traffic Our measure of port traffic is defined as the sum of the net tonnage of each ship moored at the port each day, multiplied by the percent of the day they spend at the port—crucially including ships that arrived prior to that day but still remained moored at port. To be more specific, if a ship remained moored at port all way without exiting, their contribution to port traffic would be 100% of their net tonnage (100% of the time they spent at the port). If a ship left at any point during that day, their net tonnage contribution would be less than 100% and instead determined by the amount of time they spent moored at port that day.

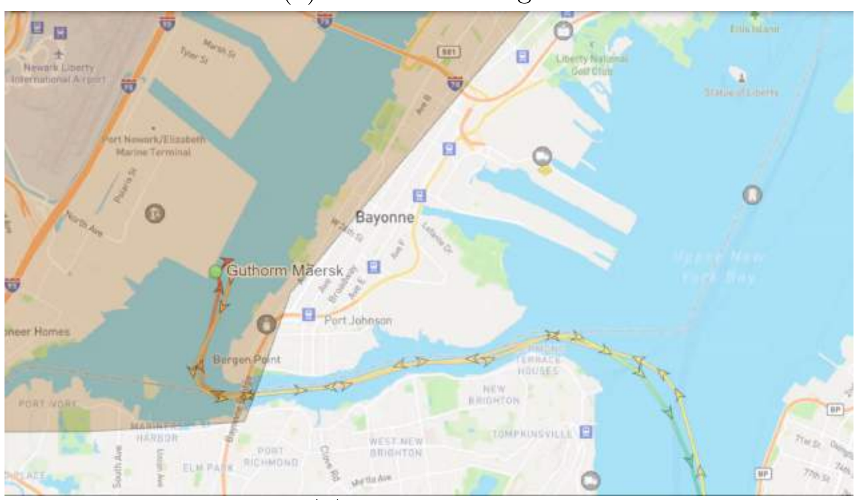
With this daily port traffic measure, we calculate moving averages of the port-level traffic for varying amounts of time. We have done this for 3, 7, 14, 21, and 28 days. We present the 28-day moving average results and have included the rest in the appendix.[4]

[4]It is acknowledged here that this measure could be interpreted as an upper bound measure of the amount of traffic at each port since using the net tonnage measure of a ship assumes that it is filled to capacity. Future work will incorporate the draft information we observe for these ships which will allow us to infer net capacity change. Additionally, an alternative lower bound measure of port traffic is a count of ships currently at the port (since smaller ships would have equal weight as large ships).

Figure 3. Illustration of AIS Mooring Paths



(a) Port of Los Angeles

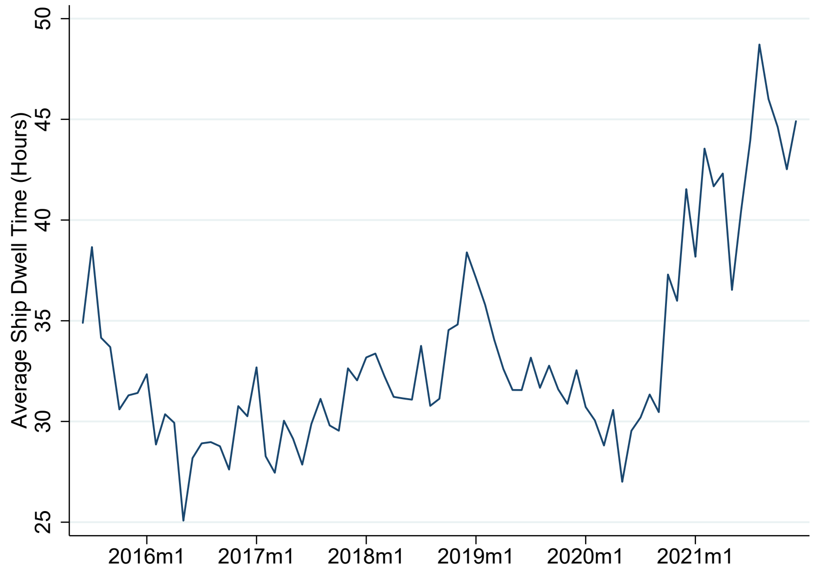


(b) Port of Newark

Notes: Panel (a) shows the containership CMA CGM Christophe Colomb at the Port of Los Angeles while Panel (b) shows the containership Guthorm Maersk at the Port of Newark. The path of each ship to and from the port shows its exact travel path. The darker regions at each port shows the port polygons as defined by the US Army Corps of Engineers.

Summary Statistics Our matched dataset from 2015 to 2021 has 3,755 unique vessels with 1,444 containerships. The top 30 ports in our dataset account for around 95% of all US container trade annually. Figure 4 plots the average of containership dwell times at the top 30 US ports from June 2015 to December 2021. The average dwell time over this period is around 33.3 hours per ship with a standard deviation of 5 hours. However, as seen in Figure 4, there is a significant increase in the ship dwell times post 2021. The average ship dwell time after 2021 is 42.8 hours.

Figure 4. Containership Dwell Times at Port



Notes: This figure plots the average of containership dwell times at the top 30 US ports from June 2015 to December 2021. Weighted by ship net tonnage.

2.2.2 Rail Dwell Times Data

We obtain weekly rail station dwell times from the Surface Transportation Board (STB). Railroads provide the STB with the average time a railcar resides at a station, measured in hours, for their 10 largest stations in terms of railcars processed. This dwell time measure excludes cars on through trains—trains that travels without stops en route. Since this dataset only captures a subset of all rail stations (albeit the largest ones), we match the ports in the previous section to their local rail stations. We do this by expanding the port polygon areas in 50km intervals. The rail stations that are captured in the buffer areas of their closest port is be considered a rail station in the vicinity of this port and is likely to service traffic to and from the port. Due to their proximity, The ports of Los Angeles and Long Beach and combined into one port for this exercise. We use a buffer area of 150km which captures 7 ports and 12 rail stations. We test the robustness of this buffer area by increasing the interval in our analysis to 200km where we capture 8 ports and 14 rail stations. Further increases to this interval result in more muted responses of rail station dwell times to port traffic, as these rail stations are much further away.

Additionally, the rail dwell times dataset is reported at the weekly level. In order to match this to our daily port traffic measure for analysis, we aggregate our port traffic measure up to the weekly level. We start our week on a Monday since we observe in our data that most ships tend to enter a port on Mondays.

Summary Statistics Figure 5 plots the average of rail station dwell times from June 2015 to December 2021. The average dwell time over this period is around 25.5 hours per station with a standard deviation of 2.5 hours. However, there is also a large decrease in dwell times around the start of the pandemic followed up a steep increase afterwards.

2.2.3 Rail Traffic Data

We have obtained access to confidential US rail traffic data from the Surface Transportation Board. This is a stratified sample of carload waybills for all U.S. rail traffic submitted by those rail carriers terminating 4,500 or more revenue carloads annually, covering 48 states (except Alaska and Hawaii). The carload waybills report the origin location, origin rail station, interchange stations, terminating station, and destination location of the freight commodities. The rich geographical information in this confidential data set allows us to study the routing of these commodities through the railroad network over this time period. Additionally, this data set also contains commodity-specific information including number of car loads, weight, freight charges, whether it is a domestic or international shipment, and its inter-modality—if the movement of this commodity included other transport modes. Overall, the data covers 48 states (except Alaska and Hawaii) and 39 STCC 2 digit commodities. Work is currently ongoing to merge both the rail traffic data to the AIS vessel traffic data at ports.

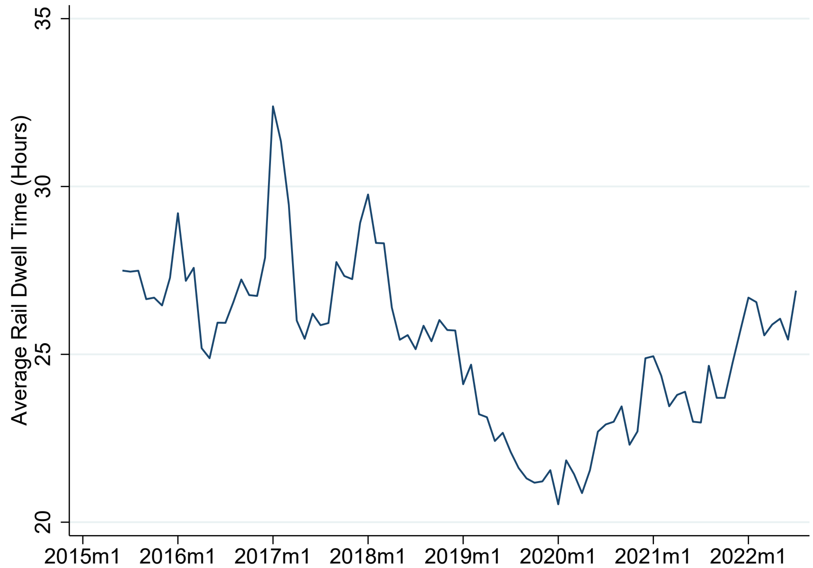
3 Economic Geography Model with Multimodal Routing

In this section, we embed multimodal routing into a standard economic geography model. We first set up the general framework, before we turn towards deriving an expression for the multimodal routing problem that can then in turn be incorporated into the equilibrium conditions and gives rise to a tractable characterization of traffic across modes.

3.1 Setup

This subsection describes a standard economic geography model with domestic trade between a discrete number of locations and freely mobile labor reallocation across locations as in Allen and Arkolakis (2014), Redding (2016), and Allen and Arkolakis (2022). In the next subsection we then embed the multimodal routing choice into this model.

Figure 5. Rail Station Dwell Times

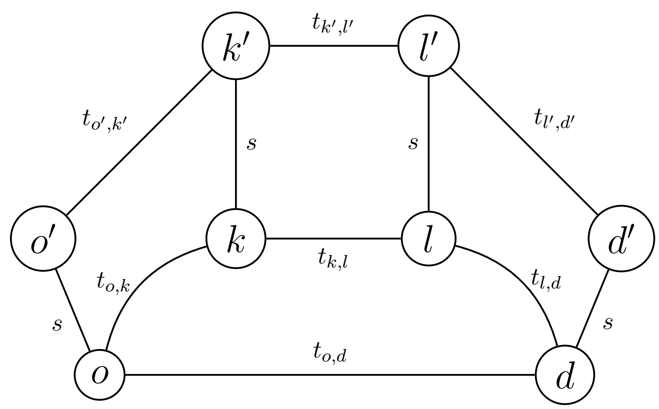


Notes: This figure plots the average time a railcar spends at a rail station from June 2015 to July 2022.

3.1.1 Geography and Transportation

Let there be a number of locations that constitute nodes on the primary transportation network ("Roads"), i.e. $k, l \in \mathbb{T} = \{1, \ldots, N_1\}$. Let there also be a number of locations that constitute nodes on the secondary transportation network ("Multimodal"), i.e. $k', l' \in \mathbb{R} = \{1, \ldots, N_2\}$. Locations on the primary network are either locations or intersections on the primary network. Locations on the secondary network constitute intersections or terminal stations. A subset of nodes on the primary and secondary network are intermodal terminals which allow for switches between the two transportation networks. Moving goods from origin i to destination j along route r, which involves a series of links index from 0 to K, is indicated by vector $r \equiv \{i = r_0, r_1, \ldots, r_K = j\}$. Since the primary road network is dense and all cities are located on it, we assume the common assumption in freight transportation—–that all routes originate and terminate on the primary network (first and last mile by road). Routes can be either unimodal or multimodal. The set of unimodal routes only use the primary network given by $\mathcal{R}^1_{ij}$. The set of multimodal routes that use both primary and secondary network, as well as the transition through intermodal terminals, is given by $\mathcal{R}^{1,2}_{ij}$. Transport costs are multiplicative, and the total cost incurred along route r from origin i to destination j is the product of the transport cost of each link along this route $\prod_{k=1}^{K} t_{r_{k-1},r_k}$. Figure 6 provides a stylized example of a graph with two distinct transportation networks.

Figure 6. Multimodal Transportation Network



Notes: The figure illustrates a simplified multimodal transport network. Nodes without primes are located on the primary road network, while nodes with primes are located on the secondary multimodal network. Each link is associated with an (iceberg) transport cost. The mode-specific transport cost of each link on either the primary or secondary network is given by t. The existence of links between nodes on both networks allows for switching between the networks—intermodal terminals (e.g. k and k' on the primary and secondary network respectively). The switching cost of going through these intermodal terminals is s.

3.1.2 Consumption, Production and Trade

A representative agent lives in location j, supplies her unit endowment of labor inelastically, earns a wage rate w_j, and purchases quantities of a continuum of consumption goods, $\nu \in [0, 1]$. She is endowed with constant elasticity of substitution (CES) preferences where the elasticity of substitution is given by $\sigma \geq 0$. Her preferences are given by,

$$U_j = \left(\sum_{\nu} q_{ij}^{\frac{\sigma-1}{\sigma}}(\nu) \right)^{\frac{\sigma}{\sigma-1}}$$

where U_j aggregates the quantities produced from all other locations i. Furthermore, we define aggregate income as Y^W, the total labor endowment as $\bar{L}$, and average per capita income as the numeraire, i.e. $Y^W/\bar{L} = 1$.

Each location i produces each good $\nu \in [0, 1]$ subject to a constant returns to scale technology and transports it to each destination j along each of the feasible routes $r \in \mathcal{R}_{ij}^1 \cup \mathcal{R}_{ij}^{1,2}$. The set of feasible routes combines unimodal and multimodal routes that connect locations on the primary network. Each shipment is subject to idiosyncratic productivity shocks, capturing uncertainties that may affect production as well as bilateral and route-specific transportation.[5] We assume

[5]Using bill of lading data for containerized imports combined with AIS vessel movement data, Ganapati, Wong and Ziv (2021) finds substantial variation in international ocean shipping routes between the United States and

perfect competition which implies that the price of good ν in destination j from origin i along route r is given by

$$p_{ij,r}(\nu) = \frac{w_i}{A_i} \frac{\prod_{k=1}^{K} t_{r_{k-1},r_k}}{\varepsilon_{ij,r}(\nu)} \equiv \frac{w_i}{A_i} \tau_{ij,r}(\nu)$$

where the marginal cost of production in i is $\frac{w_i}{A_i}$, local wages are w_i, and each worker can produce A_i units of goods. Trade cost is route-specific $\tau_{ij,r}(\nu)$ and multiplicative over all links along route r. In this context, it is assumed that individuals choose the good with the lowest price, i.e. they choose the cheapest location-route combination. Following Eaton and Kortum (2002), we assume that $\varepsilon_{ij,r}(\nu)$ is iid Fréchet distributed across routes and goods with scale parameter $1/A_i$, where A_i captures origin-specific efficiency (the same A_i as earlier) and shape parameter θ regulates the inverse of shock dispersion. Given the preference shocks, the probability that j purchases a good from i using route r is given by,[6]

$$\pi_{ij,r} = \frac{(w_i/A_i)^{-\theta} \left(\prod_{l=1}^{K} t_{r_{l-1},r_l}^{-\theta}\right)}{\sum_{k\in\mathcal{N}} (w_k/A_k)^{-\theta} \sum_{r'\in\mathcal{R}_{kj}^{1}\cup\mathcal{R}_{kj}^{1,2}} \prod_{l=1}^{K} t_{r'_{l-1},r'_l}^{-\theta}}.$$

The expected transport cost from origin i to destination j over the multimodal transportation network τ_{ij} is then given by,

$$\begin{aligned}
\tau_{ij} &= \int\limits_{\mathcal{R}_{ij}^{1}\cup\mathcal{R}_{ij}^{1,2}} \tau_{ij,r}(\nu)dr = \int\limits_{\mathcal{R}_{ij}^{1}} \tau_{ij,r}(\nu)dr + \int\limits_{\mathcal{R}_{ij}^{1,2}} \tau_{ij,r}(\nu)dr \\
&= \Gamma\left(\frac{\theta-1}{\theta}\right)\left(\underbrace{\sum_{r'\in\mathcal{R}_{ij}^{1}}\left(\prod_{k=1}^{K} t_{r'_{k-1},r'_k}\right)^{-\theta}}_{\text{Paths on road network}} + \underbrace{\sum_{r'\in\mathcal{R}_{ij}^{1,2}}\left(\prod_{k=1}^{K} t_{r'_{k-1},r'_k}\right)^{-\theta}}_{\text{Paths on multimodal network}}\right)^{-\frac{1}{\theta}}
\end{aligned} \tag{1}$$

where we are able to write the expected transport cost as the sum of the paths on the road and multimodal network because both of these sets of paths are made up of completely separable sets. Notice that this implies transport mode shares that are a function of the agent's route

each trading partner—providing empirical verification for this assumption. In the context of the transportation literature, this assumption leads to the *Stochastic User Equilibrium* (cp. Boyles, Lownes and Unnikrishnan (2021) for an exposition), where it is argued that instead of having perfect knowledge of the travel time across the whole network, agents might have imperfect knowledge and might therefore deviate from the least cost travel path. Stochastic deviations then reflect perceptions errors about travel cost rather than productivity or transport cost shocks.

[6] See detailed derivations in Appendix A.1.

choice. Specifically, we can define the share of goods transported over each mode as follows,

$$\pi_{ij}^{1} = \frac{(w_i/A_i)^{-\theta} \sum_{r' \in \mathcal{R}_{kj}^{1}} \left(\prod_{l=1}^{K} t_{r_{t-1},r_l}^{-\theta} \right)}{\sum_{k \in \mathcal{N}} (w_k/A_k)^{-\theta} \sum_{r' \in \mathcal{R}_{kj}^{1} \cup \mathcal{R}_{kj}^{1,2}} \prod_{l=1}^{K} t_{r'_{i-1},r'_l}^{-\theta}}, \; \pi_{ij}^{1,2} = \frac{(w_i/A_i)^{-\theta} \sum_{r' \in \mathcal{R}_{kj}^{1,2}} \left(\prod_{l=1}^{K} t_{r_{t-1},r_l}^{-\theta} \right)}{\sum_{k \in \mathcal{N}} (w_k/A_k)^{-\theta} \sum_{r' \in \mathcal{R}_{kj}^{1} \cup \mathcal{R}_{kj}^{1,2}} \prod_{l=1}^{K} t_{r_{i-1},r'_l}^{-\theta}}.$$

where π_{ij}^{1} describes the probability that a good that is sourced from origin i and shipped to destination j travels along the primary network only, while π_{ij}^{2} describes the probability that the good travels along the secondary multimodal network. This allows us to derive the trade flows from i to j in terms of the amount of trade along the primary and secondary network, as follows:

$$X_{ij} = \pi_{ij}^{1} E_j + \pi_{ij}^{1,2} E_j = \frac{\left(\tau_{ij}^{1}\right)^{-\theta} (w_i/A_i)^{-\theta}}{\sum_{k \in \mathcal{N}} \tau_{kj}^{-\theta} (w_k/A_k)^{-\theta}} E_j + \frac{\left(\tau_{ij}^{1,2}\right)^{-\theta} (w_i/A_i)^{-\theta}}{\sum_{k \in \mathcal{N}} \tau_{kj}^{-\theta} (w_k/A_k)^{-\theta}} E_j \tag{2}$$

where E_j is the total expenditure at location j, and the unimodal and multimodal transports expected transports can be defined as follows,

$$\tau_{ij}^{1} \equiv \left(\sum_{r' \in \mathcal{R}_{ij}^{1}} \left(\prod_{l=1}^{K} t_{r_{l-1},r_l}^{-\theta} \right) \right)^{-\frac{1}{\theta}}, \qquad \tau_{ij}^{1,2} \equiv \left(\sum_{r' \in \mathcal{R}_{ij}^{1,2}} \left(\prod_{l=1}^{K} t_{r_{l-1},r_l}^{-\theta} \right) \right)^{-\frac{1}{\theta}}$$

We then combine the uni- and multi-modal transport costs resulting in overall transport costs

$$(\tau_{ij})^{-\theta} \equiv \left(\tau_{ij}^{1}\right)^{-\theta} + \left(\tau_{ij}^{1,2}\right)^{-\theta} \tag{3}$$

which implies that the transportation cost is additively separable between the uni- and multi-modal transport cost. This setup extends the model in Allen and Arkolakis (2022) to allow for a tractable description of mode choice. Mode choice in this setting simply arises because it is *implied by* route choice. As in the original paper, the individual still chooses both a location and route to source each good. However, since routes can now be either unimodal (i.e. only relying on the primary network) or multimodal (i.e. tracing a path on both the primary and secondary network), a route choice now also implies a transport mode choice. As we will show below, this modeling choice retains the tractability of the problem, but allows us to consider considerably more complex and realistic transportation settings where agents choose between multiple modes subject to switching costs.[7]

[7] One drawback of the current setting is that the trade, route and mode choice elasticity are all pinned down by the dispersion parameter, θ. In principle, this can be relaxed by introducing a nested choice and we are currently working on such an extension.

3.2 Market Access, Gravity and Equilibrium

Before defining and deriving the equilibrium conditions, we reformulate the expression for bilateral trade flows (5) using market access terms (Anderson and van Wincoop, 2003; Redding and Venables, 2004). We follow the standard procedure and impose firstly, that good markets clear, i.e. total income in a location, Y_i, is equal to its total sales, and secondly, that trade is balanced, i.e. total expenditure, E_i, is equal to total expenditures in each location: is equal to its total purchases:

$$Y_i = \sum_{j=1}^{N} X_{ij}, \qquad E_i = \sum_{j=1}^{N} X_{ji} \tag{4}$$

This allows us to rewrite the gravity equation using market access terms, i.e.

$$X_{ij} = \left(\tau_{ij}^{1}\right)^{-\theta} \times \frac{Y_i}{\Pi_i^{-\theta}} \times \frac{E_j}{P_j^{-\theta}} + \left(\tau_{ij}^{1,2}\right)^{-\theta} \times \frac{Y_i}{\Pi_i^{-\theta}} \times \frac{E_j}{P_j^{-\theta}} \tag{5}$$

where the first term represents good flows on the primary network and the second term represents flows utilizing a multimodal path. Furthermore, the producer and consumer price index are given respectively by,

$$\Pi_i \equiv \left(\sum_{j=1}^{N} \tau_{ij}^{-\theta} E_j P_j^{\theta}\right)^{-\frac{1}{\theta}} = A_i L_i Y_i^{-\frac{\theta+1}{\theta}}, \qquad P_j = \left(\sum_{i=1}^{N} \tau_{ij}^{-\theta} Y_i \Pi_i^{\theta}\right)^{-\frac{1}{\theta}} \tag{6}$$

To derive the equilibrium conditions, we impose welfare equalization, i.e. $W_j = \frac{w_j}{P_j} u_j$ (Allen and Arkolakis, 2014) and assume localized productivity (A_i) and amenity spillovers (u_i) that depend on the density of workers in a locality, i.e.

$$A_i = \bar{A}_i L_i^{\alpha}, \qquad u_i = \bar{u}_i L_i^{\beta} \tag{7}$$

where $\bar{A}_i$ is exogenous component of productivity at location i and α determines the extent to which productivity is affected by the local population L_i (productivity spillovers), $\bar{u}_i$ is the exogenous utility derived from living in location i and β governs the extent to which amenities are affected by the location population (amenity spillovers).

To obtain the equilibrium conditions we impose balanced trade, welfare equalization, and we combine the parameterization of the local spillovers (7), the expression for trade flows (5) and the market clearing conditions (4). We then obtain the equilibrium condition:

$$\bar{A}_i^{-\theta} y_i^{1+\theta} l_i^{-\theta(1+\alpha)} = \chi \sum_{j=1}^{N} \left(\tau_{ij}^{1}\right)^{-\theta} \bar{u}_j^{\theta} y_j^{1+\theta} l_j^{\theta(\beta-1)} + \chi \sum_{j=1}^{N} \left(\tau_{ij}^{1,2}\right)^{-\theta} \bar{u}_j^{\theta} y_j^{1+\theta} l_j^{\theta(\beta-1)} \tag{8}$$

$$\bar{u}_i^{-\theta} y_i^{-\theta} l_i^{\theta(1-\beta)} = \chi \sum_{j=1}^{N} \left(\tau_{ij}^{1}\right)^{-\theta} \bar{A}_j^{\theta} y_j^{-\theta} l_j^{\theta(\alpha+1)} + \chi \sum_{j=1}^{N} \left(\tau_{ij}^{1,2}\right)^{-\theta} \bar{A}_j^{\theta} y_j^{-\theta} l_j^{\theta(\alpha+1)} \tag{9}$$

where we have written the equilibrium condition in terms of shares of world income in location i, $y_i \equiv \frac{Y_i}{Y^W}$, and shares of total labor in location i, $l_i \equiv \frac{L_i}{L^W}$. Furthermore, $\chi \equiv \left(\frac{L(\alpha+\beta)}{\bar{W}}\right)^{\theta}$ is an endogenous scalar that is inversely related to the global welfare of the spatial economy.[8] The equilibrium system is identical to the one in Allen and Arkolakis (2022), but distinguishes between trade on the primary and secondary transportation system. Specifically, given the fundamentals $\left\{\bar{A}_i, \bar{u}_i, \tau_{ij}\right\}$, the system of $2N$ equations can be solved for the $2N$ endogenous equilibrium values, $\{y_i, l_i\}$. Transportation cost is endogenous and depends on the agent's routing choice which itself depends on the underlying multimodal transportation network.

3.3 Multimodal Routing and Congestion

In this subsection we incorporate multimodal routing in a tractable manner into the spatial equilibrium. To do so we proceed in three steps. In the next subsection 3.3.1 we use the properties of partitioned matrices to obtain a tractable description of the transport cost on the multimodal network. In the following subsection 3.3.2, we characterize the traffic on the primary network, before turning towards traffic on the secondary network, and then specifying traffic at terminal stations. In the final subsection 3.3.4, we characterize the equilibrium in terms of traffic along the primary and secondary network.

3.3.1 Multimodal Routing and Transportation Cost

In the previous section, we outlined a route choice problem on a high-dimensional partitioned multimodal graph. In this subsection we use results from matrix algebra to derive an analytically convenient characterization of the resulting transportation cost in terms of the underlying adjacency matrices.[9] By explicitly enumerating all possible routes, equation (1) can be written in matrix notation as follows:

[8] See Online Appendix B.1 for detailed derivations.

[9] Our characterization is consistent with, but extends the analysis in Fan and Luo (2020) by exploiting the partitioned structure of the aggregate infrastructure matrix to find closed-form expressions for the flows on the separate parts of the network. In Online Appendix B.3 we provide a more explicit comparison.

$$\tau_{ij}^{-\theta} = \left(\sum_{K=0}^{\infty} \left(\left(\sum_{K=0}^{\infty} \mathbf{A}_1^K \right) \left(\mathbf{S} \left(\sum_{K=0}^{\infty} \mathbf{A}_2^K \right) \mathbf{S}' \right) \right)^K \left(\sum_{K=0}^{\infty} \mathbf{A}_1^K \right) \right)_{ij} \tag{10}$$

where $\mathbf{A_1} = [a_{ij}] = \left[t_{ij}^{-\theta}\right]$ is the $N^1 \times N^1$ adjacency matrix for the primary transportation network, $\mathbf{A_2} = [a_{i'j'}] = \left[t_{i'j'}^{-\theta}\right]$ is the $N^2 \times N^2$ adjacency matrix for the secondary transportation network, and $\mathbf{S} = [s_{ii'}]$ is the diagonal matrix that represents linkages between the primary and secondary transportation network. The first term in equation (10) summarizes paths that originate on the primary road network at location i and involve an arbitrary number of switches between the primary and secondary network, while the final term summarizes path that utilize the road network to reach the final destination j. These first and last terms capture the well known first and last mile problem where the dense primary network is being used to achieve final delivery in freight transportation (Rodrigue, 2020; Ranieri et al., 2018).

As long as the spectral radius of $\mathbf{A}_1$ and $\mathbf{A}_2$ is less than one (Jorgenson, Bear and Wagner, 1962; Bell, 1995), the geometric sum can be expressed as:

$$\sum_{K=0}^{\infty} \mathbf{A}_1^K = (\mathbf{I} - \mathbf{A}_1)^{-1} \equiv \mathbf{B} \qquad \sum_{K=0}^{\infty} \mathbf{A}_2^K = (\mathbf{I} - \mathbf{A}_2)^{-1} \equiv \mathbf{C} \tag{11}$$

We can furthermore define the matrix that adjusts the transport cost along the secondary transportation network for switching costs,

$$\mathbf{D} \equiv \mathbf{S} \left(\sum_{K=0}^{\infty} \mathbf{A}_2^K \right) \mathbf{S}' \tag{12}$$

Applying these definitions to equation (10), we can simplify the expression and obtain,

$$\tau_{ij}^{-\theta} = \left(\left(\sum_{K=0}^{\infty} (\mathbf{BD})^K \right) \mathbf{B} \right)_{ij} \tag{13}$$

which can be more succinctly represented by invoking the recursive formula for the inverse of a sum of matrices[10]

$$\sum_{K=0}^{\infty} (\mathbf{BD})^K \mathbf{B} = (\mathbf{B}^{-1} - \mathbf{D})^{-1} \equiv \mathbf{E} \tag{14}$$

[10] Let A and B be conformable square matrices, then an application of the Sherman–Morrison–Woodbury (Horn and Johnson, 2012) formula implies,

$$(\mathbf{A} - \mathbf{B})^{-1} = \mathbf{A}^{-1} + \mathbf{A}^{-1}\mathbf{B}(\mathbf{A} - \mathbf{B})^{-1}$$

where $\mathbf{E} = [e_{ij}]$ is the inverse of the Schur complement of the partitioned infrastructure matrix.[11] We can therefore write,

$$\tau_{ij} = e_{ij}^{-\frac{1}{\theta}} \tag{16}$$

Equation (16) directly relates the expected transportation cost that arises from the least-cost route choice problem to the underlying transport infrastructure. Specifically, it provides an analytical relationship between the primary, secondary, and switching cost matrix to the expected bilateral transportation cost, $\{\tau_{ij}\}_{i,j\in\mathcal{N}^2}$, connecting all locations on the primary network.[12]

While equation (16) is analytically convenient for us, particularly in terms of deriving a tractable expression of the general equilibrium equation in terms of primary and secondary traffic (see below), it is not directly interpretable. Applying the formula for the inverse of the partitioned matrix directly to the matrix that represents the multimodal transport infrastructure[13] gives a more intuitive representation of the overall transportation cost, i.e.

$$\begin{aligned}\tau_{ij}^{-\theta} &= \left[\mathbf{B} + \mathbf{BS}\left((\mathbf{I}-\boldsymbol{\Omega})/(\mathbf{I}-\mathbf{A_1})\right)^{-1}\mathbf{S}'\mathbf{B}\right]_{ij} \\ &= \left(\tau_{ij}^{1}\right)^{-\theta} + \left(\tau_{ij}^{1,2}\right)^{-\theta}\end{aligned} \tag{17}$$

where $(\mathbf{I}-\boldsymbol{\Omega})/(\mathbf{I}-\mathbf{A_1}) \equiv (\mathbf{I}-\mathbf{A_2}) - \mathbf{SBS}'$ defines the Schur complement of the matrix $(\mathbf{I}-\mathbf{A_2})$ in $(\mathbf{I}-\boldsymbol{\Omega})$. The expression intuitively decomposes the overall transport cost that arises from the

which has a recursive structure where we can solve for a geometric sum expression,

$$(\mathbf{A}-\mathbf{B})^{-1} = \sum_{k=0}^{\infty}\left(\mathbf{A}^{-1}\mathbf{B}\right)^{k}\mathbf{A}^{-1}$$

[11] More precisely (16) represents the expected transportation cost as the Schur complement of the identity minus the adjacency matrix of the secondary infrastructure network in the partitioned aggregate infrastructure matrix, i.e.

$$\mathbf{E} \equiv \left(\mathbf{B}^{-1} - \mathbf{D}\right)^{-1} = (\mathbf{I}-\boldsymbol{\Omega})/\left(\mathbf{I}-\mathbf{A}_2\right) \tag{15}$$

where $(\mathbf{I}-\boldsymbol{\Omega})/\left(\mathbf{I}-\mathbf{A}_2\right)$ is defined as the Schur complement of the matrix $(\mathbf{I}-\mathbf{A_2})$ in $(\mathbf{I}-\boldsymbol{\Omega})$.

[12] While we have presented the problem with agents residing on the primary network, in principle similar calculations would allow us to obtain the transportation cost from and to locations on the secondary network.

[13] We can construct a block matrix that represents the overall adjacency matrix of the multimodal network by arranging the adjacency matrix of the primary and secondary network on the block diagonal and by incorporating switching points between primary and secondary network by positioning the $\mathbf{S}$ matrix on the off-diagonal block, i.e.

$$\Omega = \begin{bmatrix} \mathbf{A_1} & \mathbf{S} \\ \mathbf{S}' & \mathbf{A_2} \end{bmatrix}$$

where as above $\mathbf{A_1} = [a_{ij}] = \left[t_{ij}^{-\theta}\right]$ is the $N^1 \times N^1$ adjacency matrix for the primary transportation network, $\mathbf{A_2} = [a_{i'j'}] = \left[t_{i'j'}^{-\theta}\right]$ is the $N^2 \times N^2$ adjacency matrix for the secondary transportation network, and $\mathbf{S} = [s_{ii'}]$ is the diagonal matrix that represents linkages between the primary and secondary transportation network. We can obtain equation (17) by applying the formula for the inverse of a partitioned matrix (Horn and Johnson, 2012) to the matrix $(\mathbf{I}-\boldsymbol{\Omega})$.

least cost routing choice into two distinct terms that mirror the decomposition in equation (3). The first term, $\mathbf{B}_{ij} = \left(\tau_{ij}^{1}\right)^{-\theta}$, summarizes the universe of *unimodal* paths and therefore simply reflects the travel cost that would arise if the agent only considered the primary network as in Allen and Arkolakis (2022). The second term, $\left(\mathbf{BS}\left(\mathbf{\Omega}/(\mathbf{I}-\mathbf{A_1})\right)^{-1}\mathbf{S}'\mathbf{B}\right)_{ij} = \left(\tau_{ij}^{1,2}\right)^{-\theta}$ traces out the additional *multimodal* paths that originate and terminate on the primary network but percolate through the secondary network. The expression calculates the cost of transitioning at any feasible point to the secondary network then routing along the secondary network and possibly transitioning back and forth between primary and secondary network and finally transitioning back to the primary network to terminate the trip in a specific location. The transitioning between secondary and primary network is embodied in the Schur complement.[14] The overall transport cost is then lowered by having the additional option of utilizing the secondary transportation network. The extent to which this is possible depends on the feasible set of routes and is determined by both (1) the extent to which the topology of the primary network allows access of the secondary network - characterized by the adjacency matrix $\mathbf{A_1}$ and the availability of switching terminals as given by $\mathbf{S}$ and (2) by the topology of the secondary network - characterized by the adjacency matrix $\mathbf{A_2}$. In the extreme case where between location i and j there are no possible routes along the secondary network, then the formula gives exactly the same transport cost as in Allen and Arkolakis (2022).

3.3.2 Modal Traffic Flows

We proceed by deriving the implied traffic flow along different parts of the network. In this subsection we will re-iterate the derivation of the traffic on the primary network as in Allen and Arkolakis (2022) before then turning towards novel results, i.e. the traffic on the secondary network and the traffic at terminal stations. The purpose of deriving these objects is to introduce traffic and congestion in the equilibrium conditions (8) and (9). Specifically, we will be interested in introducing switching costs that depend on the throughput at any given terminal, thus creating

[14] If switching costs are sufficiently costly to make routes with multiple switches between the first and secondary transportation network prohibitively costly, then this expression can be simplified and we can replace the Schur complement with the transportation cost on the secondary network, and equation (17) becomes,

$$
\begin{aligned}
\tau_{ij}^{-\theta} &= \left[\mathbf{B} + \mathbf{BS}\left(\mathbf{I}-\mathbf{A_2}\right)^{-1}\mathbf{S}'\mathbf{B}\right]_{ij} \\
&= \left(\tau_{ij}^{1}\right)^{-\theta} + \left(\tau_{ij}^{1,2}\right)^{-\theta}
\end{aligned} \tag{18}
$$

where now the multimodal transport cost simply depends on the access to the secondary transport network and expected transport cost along the secondary transport network.

bottlenecks in the multimodal transportation system.

Traffic on the Primary Network. We begin by characterizing the traffic on the primary network.[15] We follow AA2022 and characterize the number of times a link (k,l) is used in trade between (i,j), π_{ij}^{kl}, which we refer to as link intensity. To obtain the link intensity we construct the probability that a route i to j is used and the number of times that a route passes through a particular link (k,l) and sum over all possible routes across the network. We obtain,

$$\pi_{ij}^{kl} \equiv \sum_{r\in\mathcal{R}_{ij}^{1}} \left(\frac{\pi_{ij,r}}{\sum_{r'\in\mathcal{R}_{ij}^{1}\cup\mathcal{R}_{ij}^{1,2}} \pi_{ij,r'}} \right) n_r^{kl} + \sum_{r\in\mathcal{R}_{ij}^{1,2}} \left(\frac{\pi_{ij,r}}{\sum_{r'\in\mathcal{R}_{ij}^{1}\cup\mathcal{R}_{ij}^{1,2}} \pi_{ij,r'}} \right) n_r^{kl}$$

Compared to the original paper we can distinguish between traffic on the primary network that arises due to unimodal and multimodal routes. Since multimodal routes utilize a combination of links both on the primary and secondary network, they also generate traffic on the primary network which needs to be accounted for. Using some matrix manipulation we obtain the same expression as AA2022,

$$\pi_{ij}^{kl} = \left(\frac{\tau_{ij}}{\tau_{ik} t_{kl} \tau_{ij}} \right)^{\theta} \tag{19}$$

where τ_{ij} is the expected transportation cost across the multimodal transport system as defined in (16). We can then use these derivations to characterize traffic on the primary network,

$$\Xi_{kl} \equiv \sum_{i\in\mathcal{N}} \sum_{j\in\mathcal{N}} \sum_{r'\in\mathcal{R}_{ij}^{1}\cup\mathcal{R}_{ij}^{1,2}} \pi_{ij,r} n_r^{kl} E_j = \sum_{i\in\mathcal{N}} \sum_{j\in\mathcal{N}} \pi_{ij}^{kl} X_{ij}, \tag{20}$$

Combining the market access (5) and the link intensity expression (19) allows us to derive the expression for equilibrium traffic,

$$\Xi_{kl}^{1} = t_{kl}^{-\theta} \times P_k^{-\theta} \times \Pi_l^{-\theta} \tag{21}$$

Equation (21) is a gravity equation for traffic on the primary network. The expression connects traffic flows to inward, $P_k^{-\theta}$ and outward market access measures, $\Pi_l^{-\theta}$. Both market access measures depend on the transportation cost across the multimodal transport network.

[15] Detailed derivations for traffic on the primary and secondary network as well as traffic at terminal stations is given in Appendix A.3.

Traffic on the Secondary Network. We proceed by characterizing the traffic on the secondary transport network. We define $\pi_{ij}^{k'l'}$ as the link intensity of a link $k'l'$ on the secondary transportation network, which is given by,

$$\pi_{ij}^{k'l'} \equiv \sum_{r \in \mathcal{R}_{ij}^{1,2}} \left(\frac{\pi_{ij,r}}{\sum_{r' \in \mathcal{R}_{ij}^{1} \cup \mathcal{R}_{ij}^{1,2}} \pi_{ij,r'}} \right) n_r^{k'l'} \tag{22}$$

which can be written using matrix algebra as,

$$\pi_{ij}^{k'l'} = \left(\frac{\tau_{ij}}{\tau_{ik} s_{kk'} \tau_{k'l'} s_{l'l} \tau_{lj}} \right)^{\theta} \tag{23}$$

The difference between (19) compared to (23) consists in tracing out the importance of linkages along the secondary network relative to the overall average transport cost between i and j. It is important to note that this not only depends on the transportation cost along the secondary network between k' and l', but also on the switching cost to the secondary network at node k and l. Using this expression for link intensity we can characterize traffic on the secondary transport infrastructure, i.e.

$$\Xi_{k'l'} \equiv \sum_{i \in \mathcal{N}} \sum_{j \in \mathcal{N}} \sum_{r \in \mathcal{R}_{ij}^{1,2}} \pi_{ij,r} n_r^{k'l'} E_j = \sum_{i \in \mathcal{N}} \sum_{j \in \mathcal{N}} \pi_{ij}^{k'l'} X_{ij},$$

Combining the market access (5) and the link intensity expression (23) allows us to derive the expression for equilibrium traffic on the secondary transportation network,

$$\Xi_{kl}^2 = s_{kk'}^{-\theta} \tau_{k'l'}^{-\theta} s_{l'l}^{-\theta} \times P_k^{-\theta} \times \Pi_l^{-\theta}, \tag{24}$$

Equation (24) is the natural counterpart to equation (21) for the secondary network. It also reflects a gravity equation and connects traffic flows to market access measures. Crucially, bilateral traffic here depends on the transportation cost on the secondary network and the switching costs incurred when transitioning from the primary to the secondary network.

Traffic at Terminals. Finally, we turn towards characterizing traffic at terminals where switches between the primary and secondary network occur. Since a terminal is represented by a link on the partitioned graph, we can simply follow the same steps as before. For the link intensity between a node on the primary network k and a node on the secondary network k',

$\pi_{ij}^{kk'}$, is as follows:

$$\pi_{ij}^{kk'} \equiv \sum_{r \in \mathcal{R}_{ij}^{1,2}} \left(\frac{\pi_{ij,r}}{\sum_{r' \in \mathcal{R}_{ij}^{1} \cup \mathcal{R}_{ij}^{1,2}} \pi_{ij,r'}} \right) n_r^{kk'}$$

With some matrix calculus we obtain,

$$\pi_{ij}^{kk'} = \left(\frac{\tau_{ij}}{\tau_{ik} s_{kk'} \tau_{k'j}} \right)^{\theta} \tag{25}$$

We can characterize equilibrium flows,

$$\Xi_{kk'} \equiv \sum_{i \in \mathcal{N}} \sum_{j \in \mathcal{N}} \sum_{r \in \mathcal{R}_{ij}^{1,2}} \pi_{ij,r} n_r^{kk'} E_j = \sum_{i \in \mathcal{N}} \sum_{j \in \mathcal{N}} \pi_{ij}^{kk'} X_{ij}, \tag{26}$$

Combining with the market access gravity equation, we obtain,

$$\Xi_{kk'} = (s_{kk'})^{-\theta} \times P_k^{-\theta} \times \sum_l \tau_{k'l'}^{-\theta} s_{l'l}^{-\theta} \Pi_l^{-\theta}, \tag{27}$$

$$\Xi_{k'k} = (s_{k'k})^{-\theta} \times \Pi_k^{-\theta} \times \sum_l \tau_{k'l'}^{-\theta} s_{l'l}^{-\theta} P_k^{-\theta}, \tag{28}$$

Equations (27) and (28) differ slightly to the previous traffic equations in that they feature an additional summation term. This summation term is a higher order market access term that reflects the fact that in- and outgoing traffic at a terminal depends on the sum of traffic that is generated by nodes that can be reached via that terminal along the secondary network. This higher order market access term is also a natural measure of the centrality of the terminals in terms of connecting primary and secondary network and thus - in a sense - their capacity to become a bottleneck to the overall transportation network.

Taking stock, equations (21) and (24) characterize traffic on the primary and secondary network, while (27) and (28) characterize equilibrium traffic at terminals.

3.3.3 Congestion

In a final step before turning towards adjusting the equilibrium equations to reflect traffic, we incorporate congestion on the primary network and at terminal stations.[16] The motivation and parameterization for congestion on the primary network follows Allen and Arkolakis (2022). We

[16]While in principle it is possible to extend the framework to allow for congestion on the secondary network, in this current iteration we only incorporate congestion on the primary network and at terminal stations. The introduction of an additional margin of congestion on the secondary network would substantially reduce tractability.

assume that the direct cost of traveling over a particular link on the primary network depends on the amount of traffic that travels through that link. Specifically, we assume,

$$t_{kl} = \bar{t}_{kl} \left(\Xi^1_{kl}\right)^{\lambda_1}, \tag{29}$$

where λ_1 determines the strength of congestion on the primary network, and $\overline{\mathbf{T}} \equiv [\bar{t}_{kl}]$ is the infrastructure network for the primary network and Ξ^1_{kl} represents the traffic on the primary network. Intuitively, as long as $\lambda_1 > 0$, this expression increase transport cost as traffic on a link increases. While somewhat different to the more commonly used Bureau of Public Roads (BPR) function (Boyles, Lownes and Unnikrishnan, 2021), it is both analytically convenient and can be micro-founded in a simple model where transportation costs are log-linear in travel time and speed is a log-linear function of traffic congestion as shown in Allen and Arkolakis (2022). This expression allows us to derive equilibrium traffic flows in terms of the fundamental transport cost of each edge. First note, that transport cost is now a function of the market access terms, i.e.

$$t_{kl} = \bar{t}_{kl}^{\frac{1}{1+\theta\lambda_1}} \times P_k^{-\frac{\theta\lambda_1}{1+\theta\lambda_1}} \times \Pi_l^{-\frac{\theta\lambda_1}{1+\theta\lambda_1}},$$

Combining this with the expression for equilibrium traffic flows on the primary network (21), we obtain,

$$\Xi^1_{kl} = \bar{t}_{kl}^{\frac{\theta}{1+\theta\lambda_1}} \times P_k^{-\frac{\theta}{1+\theta\lambda_1}} \times \Pi_l^{-\frac{\theta}{1+\theta\lambda_1}} \tag{30}$$

where now overall traffic depends on the inward and outward market access terms, the fundamental transport capacity of each link, as well as the strength of the congestion externality, λ_1. Intuitively, as better market access improves traffic flow on each link this also increases congestion. Therefore the impact taking congestion into account is somewhat muted.

Secondly, we introduce congestion at terminals. We assume that the direct cost of transiting through a terminal depends on the overall traffic at the terminal, i.e.

$$s_{kk'} = \bar{s}_{kk'} \left(\Xi^2_{kk'}\right)^{\lambda_2}, \tag{31}$$

$$s_{k'k} = \bar{s}_{k'k} \left(\Xi^2_{k'k}\right)^{\lambda_2}, \tag{32}$$

where λ_2 determines the strength of congestion at terminals, and $\overline{\mathbf{S}} \equiv [\bar{s}_{kk'}]$ is the switching matrix that connects primary and secondary network and $\Xi^2_{kk'}$ represents the traffic at the terminal location that is transitioning from the primary to the secondary network, and $\Xi^2_{k'k}$ represents the traffic that is transitioning from the secondary to the primary network. The first equation determines the transport cost of traffic from the primary to the secondary network, while, symmetrically, the second equation determines the transportation cost for routes that at location k transition from the secondary to the primary network. Intuitively, as long as $\lambda_2 > 0$, this expression increases transportation cost as traffic at the terminal location increases.

Combining both (31) and (32) with the expression for equilibrium traffic at terminal stations, (28) and (27), we obtain the expression for equilibrium switching costs at terminals, i.e.

$$s_{k'k} = \bar{s}_{k'k}^{\frac{1}{1+\theta\lambda_2}} \times \Pi_k^{-\frac{\theta\lambda_2}{1+\theta\lambda_2}} \times \left(\sum_l \tau_{k'l'}^{-\theta} s_{l'l}^{-\theta} P_l^{-\theta} \right)^{-\frac{\lambda_2}{1+\theta\lambda_2}} \tag{33}$$

$$s_{kk'} = \bar{s}_{kk'}^{\frac{1}{1+\theta\lambda_2}} \times P_k^{-\frac{\theta\lambda_2}{1+\theta\lambda_2}} \times \left(\sum_l \tau_{k'l'}^{-\theta} s_{l'l}^{-\theta} \Pi_l^{-\theta} \right)^{-\frac{\lambda_2}{1+\theta\lambda_2}} \tag{34}$$

which implies equilibrium traffic flows on the secondary transportation network, given by,

$$\Xi_{kl}^2 = \bar{s}_{kk'}^{-\frac{\theta}{1+\theta\lambda_2}} \times P_k^{-\frac{\theta}{1+\theta\lambda_2}} \times \left(\sum_l \tau_{k'l'}^{-\theta} s_{l'l}^{-\theta} \Pi_l^{-\theta} \right)^{-\frac{\theta\lambda_2}{1+\theta\lambda_2}} \times \bar{s}_{ll'}^{-\frac{\theta}{1+\theta\lambda_2}} \times \Pi_l^{-\frac{\theta}{1+\theta\lambda_2}} \times \left(\sum_k \tau_{l'k'}^{-\theta} s_{k'k}^{-\theta} \Pi_k^{-\theta} \right)^{-\frac{\theta\lambda_2}{1+\theta\lambda_2}} \times \tau_{k'l'}^{-\theta} \tag{35}$$

as well as equilibrium traffic flows at terminals, i.e.

$$\Xi_{kk'}^2 = \bar{s}_{kk'}^{-\frac{\theta}{1+\theta\lambda_2}} \times P_k^{-\frac{\theta}{1+\theta\lambda}} \times \left(\sum_l \tau_{k'l'}^{-\theta} s_{l'l}^{-\theta} \Pi_l^{-\theta} \right)^{\frac{1+2\theta\lambda_2}{1+\theta\lambda_2}} \tag{36}$$

$$\Xi_{k'k}^2 = \bar{s}_{k'k}^{-\frac{\theta}{1+\theta\lambda_2}} \times \Pi_k^{-\frac{\theta}{1+\theta\lambda_2}} \times \left(\sum_l \tau_{k'l'}^{-\theta} s_{l'l}^{-\theta} P_l^{-\theta} \right)^{\frac{1+2\theta\lambda_2}{1+\theta\lambda_2}} \tag{37}$$

Taking stock, (30) develops the traffic that arises once congestion on the primary network is taken into account, while (35), (37) and (36) characterizes traffic on the secondary network and at terminal stations once congestion costs at terminals are taken into account. Traffic is generally increasing in market access which is properly to be understood as market access across the primary and secondary transportation network. Terminals can become important bottlenecks and congestion at terminals can lower the attractiveness of multimodal paths and thus traffic on the secondary network, as is apparent by equation (35). Changes in switching costs therefore imply modal substitution and the elasticity is given by $\frac{\partial \ln \Xi_{k1}^2}{\partial \ln \bar{s}_{kk'}} = -\frac{\theta}{1+\theta\lambda_2}$ where the net effect is determined by the counter-acting forces of congestion (λ_2) relative to the strength of route (and therefore modal) substitution (θ).

3.3.4 General Equilibrium with Road and Rail Traffic

In a final step, we re-consider the equilibrium equations (8) and (9) which pin down income, welfare and labor densities as a function of the transportation cost. Our framework determines transportation cost endogenously as a function of the routing (and therefore mode choice) of the agent, subject to congestion forces along the primary network and at terminal stations. In this section we combine the equilibrium condition with the expression for the endogenous transportation cost (16) and perform a matrix inversion to obtain the equilibrium in terms of primary and secondary traffic[17]. This matrix inversion gives the following equations:[18]

$$\begin{aligned} y_i^{\frac{1+\theta\lambda_1+\theta}{1+\theta\lambda_1}} l_i^{\frac{-\theta(1+\alpha+\theta\lambda_1(\alpha+\beta))}{1+\theta\lambda_1}} &= \chi \bar{A}_i^\theta \bar{u}_i^\theta y_i^{\frac{1+\theta\lambda_1+\theta}{1+\theta\lambda_1}} l_i^{\frac{\theta(\beta-1)}{1+\theta\lambda_1}} \\ &+ \chi^{\frac{\theta\lambda_1}{1+\theta\lambda_1}} \sum_j \left(\bar{t}_{ij}\bar{L}^{\lambda_1}\right)^{-\frac{\theta}{1+\theta\lambda_1}} \bar{A}_i^\theta \bar{u}_i^{\theta\frac{\theta\lambda_1}{1+\theta\lambda_1}} \bar{A}_j^{-\frac{\theta}{1+\theta\lambda_1}} y_j^{\frac{1+\theta}{1+\theta\lambda_1}} l_j^{-\frac{\theta(1+\alpha)}{1+\theta\lambda_1}} \\ &+ \sum_j s_{ii'}^{-\theta}\tau_{i'j'}^{-\theta}s_{j'j}^{-\theta}\bar{A}_j^{-\theta}y_j^{1+\theta}l_j^{-\theta(1+\alpha)}\bar{A}_i^\theta l_i^{-\theta(\beta-1)\frac{\theta\lambda_1}{1+\theta\lambda_1}} y_i^{-\theta\frac{\theta\lambda_1}{1+\theta\lambda_1}} \end{aligned} \tag{38}$$

$$\begin{aligned} y_i^{-\frac{\theta(1-\lambda_1)}{1+\theta\lambda_1}} l_i^{\frac{\theta(1-\beta-\theta\lambda_1(\alpha+\beta))}{1+\theta\lambda_1}} &= \chi \bar{A}_i^\theta \bar{u}_i^\theta y_i^{-\frac{\theta(1-\lambda_1)}{1+\theta\lambda_1}} l_i^{\frac{\theta(\alpha+1)}{1+\theta\lambda_1}} \\ &+ \chi^{\frac{\theta\lambda_1}{1+\theta\lambda_1}} \sum_j \left(\bar{t}_{ji}\bar{L}^{\lambda_1}\right)^{-\frac{\theta}{1+\theta\lambda_1}} \bar{A}_i^{\theta\frac{\theta\lambda_1}{1+\theta\lambda_1}} \bar{u}_i^\theta \bar{u}_j^{-\frac{\theta}{1+\theta\lambda_1}} l_j^{\frac{\theta(1-\beta)}{1+\theta\lambda_1}} y_j^{-\frac{\theta}{1+\theta\lambda_1}} \\ &+ \sum_j s_{jj'}^{-\theta}\tau_{j'i'}^{-\theta}s_{i'i}^{-\theta}\bar{u}_j^{-\theta}y_j^{-\theta}l_j^{\theta(1-\beta)}\bar{u}_i^\theta l_i^{-\theta(1+\alpha)\frac{\theta\lambda_1}{1+\theta\lambda_1}} y_i^{\frac{\theta\lambda_1(1+\theta)}{1+\theta\lambda_1}} \end{aligned} \tag{39}$$

Equations (38) and (39) describe the equilibrium distribution of economic activity as function of the underlying $\{y_i, l_i\}$ as a function of the model parameters, $\{\alpha, \beta, \theta, \lambda\}$, geography and underlying $\{\bar{A}_i, \bar{u}_i\}$, as well as the fundamental transportation infrastructure, i.e. the primary transport network, $\overline{\mathbf{T}} \equiv [\bar{t}_{kl}]$, the terminal transport network connecting primary and secondary network, $\overline{\mathbf{S}} \equiv [\bar{S}_{kk'}]$, as well as the secondary transport network $\overline{\mathbf{T}} \equiv [\bar{t}_{kl}]$.

While the introduction of multimodal transportation on a segmented transportation system introduced added conceptual complexity to the problem, the equilibrium system remains *as tractable* as the system presented in Allen and Arkolakis (2022). Specifically, the convenience

[17] Detailed derivations in Appendix A.4

[18] For simplicity we do not yet substitute at this point for how switching costs depend on traffic at terminal locations as mirrored by equations (28) and (27). We will do so in the next step when deriving the counterfactual equilibrium.

of the additive separability of the inverse of a partitioned matrix allows us to derive a similar equilibrium system as the original one, with the only alteration being an added summation term in each equation that traces out the spatial variation in access to the secondary transport system. The spatial distribution of economic activity is then jointly determined by the topography of the primary and secondary transport system.

3.4 Counterfactuals

To evaluate the welfare impact of infrastructure investments along either the primary, secondary network or terminal locations in a setting where agents make complicated routing and mode choices while also allowing for a rich characterization of congestion across the multimodal transport system as outlined in 3.3.3. To do so we first adjust the equilibrium conditions, (38) and (39), to reflect the congestion forces outlined above. We then follow Dekle, Eaton and Kortum (2008) and employ 'Hat Algebra' to express the equilibrium in terms of changes of the endogenous variables. In the following we denote with hats changes in variables, $\hat{\gamma}_i \equiv \frac{\gamma_i'}{\gamma_i}$.[19] The proposition below describes the resulting system of equations that determines the counterfactual equilibrium.

Proposition 1 (Counterfactual Equilibrium) *Consider an economy in equilibrium with primary transport network, $\overline{\mathbf{T}} \equiv [\bar{t}_{kl}]$, and a terminal transport network connecting primary and secondary network, $\overline{\mathbf{S}} \equiv \left[\bar{S}_{kk'}\right]$, as well as the secondary transport network $\overline{\mathbf{T}} \equiv [\bar{t}_{kl}]$. Consider any change either in the underlying infrastructure network denoted by $\hat{\bar{t}}_{kl}$, or any change in the switching cost, $\hat{\bar{s}}_{kk'}$. Given observed traffic flows $\left(\Xi^1_{ij}, \Xi^2_{i'j'}\right)$, economic activity in the geography (Y_i, E_j), and parameters $\{\alpha, \beta, \theta, \lambda_1, \lambda_2, \nu\}$, the equilibrium change in economic outcomes $\left(\hat{y}_i, \hat{l}_i, \hat{\chi}\right)$ is the solution of the following system of equations:*

$$
\begin{aligned}
\hat{l}_i^{\frac{-\theta(1+\alpha+\theta\lambda_1(\alpha+\beta))}{1+\theta\lambda_1}} \hat{y}_i^{\frac{1+\theta\lambda_1+\theta}{1+\theta\lambda_1}} = {} & \hat{\chi}\left(\frac{E_i}{E_i + \sum_j \Xi^1_{ij} + \sum_j \Xi^2_{ij}}\right) \hat{y}_i^{\frac{1+\theta\lambda_1+\theta}{1+\theta\lambda_1}} \hat{l}_i^{\frac{\theta(\beta-1)}{1+\theta\lambda_1}} \\
& + \hat{\chi}^{\frac{\theta\lambda_1}{1+\theta\lambda_1}} \sum_j \left(\frac{\Xi^1_{ij}}{E_i + \sum_j \Xi^1_{ij} + \sum_j \Xi^2_{ij}}\right) \hat{t}_{ij}^{-\frac{\theta}{1+\theta\lambda_1}} \hat{l}_j^{\frac{-\theta(\alpha+1)}{1+\theta\lambda_1}} \hat{y}_j^{\frac{\theta+1}{1+\theta\lambda_1}} \\
& + \hat{\chi}^{\frac{2\theta\lambda_2}{1+\theta\lambda_2}} \left(\hat{y}_i \hat{l}_i^{\beta-1}\right)^{\frac{\theta^2(\lambda_1-\lambda_2)}{(1+\theta\lambda_1)(1+\theta\lambda_2)}} \sum_j \left(\frac{\Xi^2_{ij}}{E_i + \sum_j \Xi^1_{ij} + \sum_j \Xi^2_{ij}}\right) \hat{\bar{s}}_{ii'}^{-\frac{\theta}{1+\theta\lambda_2}} \hat{\tau}_{i'j'}^{-\theta} \hat{\bar{s}}_{j'j}^{-\frac{\theta}{1+\theta\lambda_2}} \hat{l}_j^{\frac{-\theta(\alpha+1)}{1+\theta\lambda_2}} \hat{y}_j^{\frac{\theta+1}{1+\theta\lambda_2}} \\
& \times \left(\sum_l \frac{\Xi^2_{i'l'}}{\sum_{l'} \Xi^2_{i'l'}} \hat{\tau}_{i'l'}^{-\theta} \hat{s}_{l'l}^{-\theta} \left(\hat{l}_l^{\alpha+1} \hat{y}_l^{-\frac{\theta+1}{\theta}}\right)^{-\theta}\right)^{-\frac{\theta\lambda_2}{1+\theta\lambda_2}} \left(\sum_l \frac{\Xi^2_{j'l'}}{\sum_{l'} \Xi^2_{j'l'}} \hat{\tau}_{j'l'}^{-\theta} \hat{s}_{l'l}^{-\theta} \left(\hat{y}_l \hat{l}_l^{\beta-1}\right)^{-\theta}\right)^{-\frac{\theta\lambda_2}{1+\theta\lambda_2}}
\end{aligned}
$$

[19] Detailed derivations are provided in Online Appendix C.1

$$
\begin{aligned}
\hat{l}_i^{\frac{-\theta(1+\alpha+\theta\lambda_1(\alpha+\beta))}{1+\theta\lambda_1}} \hat{y}_i^{\frac{-\theta(1-\lambda_1)}{1+\theta\lambda_1}} &= \hat{\chi}\left(\frac{Y_i}{Y_i+\sum_j \Xi^1_{ji}+\sum_j \Xi^2_{ji}}\right) \hat{y}_i^{\frac{-\theta(1-\lambda_1)}{1+\theta\lambda_1}} \hat{l}_i^{\frac{\theta(\alpha+1)}{1+\theta\lambda_1}} \\
&+ \hat{\chi}^{\frac{\theta\lambda_1}{1+\theta\lambda_1}} \sum_j \left(\frac{\Xi^1_{ij}}{Y_i+\sum_j \Xi^1_{ji}+\sum_j \Xi^2_{ji}}\right) \hat{\bar{t}}_{ji}^{-\frac{\theta}{1+\theta\lambda_1}} \hat{l}_j^{\frac{\theta(1-\beta)}{1+\theta\lambda_1}} \hat{y}_j^{-\frac{\theta}{1+\theta\lambda_1}} \\
&+ \hat{\chi}^{\frac{2\theta\lambda_2}{1+\theta\lambda_2}} \left(\hat{l}_i^{\alpha+1}\hat{y}_i^{-\frac{\theta+1}{\theta}}\right)^{\frac{\theta^2(\lambda_1-\lambda_2)}{(1+\theta\lambda_1)(1+\theta\lambda_2)}} \sum_j \left(\frac{\Xi^2_{ij}}{Y_i+\sum_j \Xi^1_{ji}+\sum_j \Xi^2_{ji}}\right) \hat{\bar{s}}_{jj'}^{-\frac{\theta}{1+\theta\lambda_2}} \hat{\tau}_{j'i'}^{-\theta} \hat{\bar{s}}_{i'i}^{-\frac{\theta}{1+\theta\lambda_2}} \hat{l}_j^{\frac{\theta(1-\beta)}{1+\theta\lambda_2}} \hat{y}_j^{-\frac{\theta}{1+\theta\lambda_2}} \\
&\times \left(\sum_l \frac{\Xi^2_{i'l'}}{\sum_{l'} \Xi^2_{i'l'}} \hat{\tau}_{i'l'}^{-\theta} \hat{s}_{l'l}^{-\theta} \left(\hat{y}_l \hat{l}_l^{\beta-1}\right)^{-\theta}\right)^{-\frac{\theta\lambda_2}{1+\theta\lambda_2}} \left(\sum_l \frac{\Xi^2_{j'l'}}{\sum_{l'} \Xi^2_{j'l'}} \hat{\tau}_{j'l'}^{-\theta} \hat{s}_{l'l}^{-\theta} \left(\hat{l}_l^{\alpha+1} \hat{y}_l^{-\frac{\theta+1}{\theta}}\right)^{-\theta}\right)^{-\frac{\theta\lambda_2}{1+\theta\lambda_2}}
\end{aligned}
$$

Proposition (1) indicates that given observed traffic flows on the primary network, bilateral flows on the secondary network[20], $(\Xi^1_{ij}, \Xi^2_{i'j'})$ as well as knowledge of the model parameters, $\{\alpha, \beta, \theta, \lambda_1, \lambda_2, \nu\}$, we can employ the model to evaluate infrastructure improvements along the primary network or at terminal stations, thus improving the connectedness of the primary and secondary transport network. The proposition provides a straightforward extension of equation (28) and (29) in AA2022. The only difference is the presence of the additional summation term at the end of each equation, which mirrors the presence and importance of the secondary transportation system. This adjustments adds a novel channel towards evaluation infrastructure improvements. In this setting, the impact of improving transportation infrastructure has the same direct and general equilibrium effect as in AA2022 where route choice is impacted, congestion can be alleviated, input and output markets can adjust and where all this adds up to welfare gains. In our setting, additionally, we also feature a direct interplay between the primary and secondary network. Mode-specific infrastructure investments can lead to modal diversion and thus alleviate congestion on the alternative transport network. The extent to which this might occur depends on the cross-sectional variation in the access to the secondary transportation system, which is reflected by variations in the weights on the final term across space.

As a corollary we can also characterize the change in the equilibrium traffic flows along the primary and secondary transport system.

Corollary 1 *Given the equilibrium change in economic outcomes* $\left(\hat{y}_i, \hat{l}_i, \hat{\chi}\right)$*, observed traffic flows* $\left(\Xi^1_{ij}, \Xi^2_{i'j'}\right)$*, economic activity in the geography* (Y_i, E_j)*, and parameters* $\{\alpha, \beta, \theta, \lambda_1, \lambda_2, \nu\}$ *, the resulting change in the traffic flows can be computed using the following formulae:*

[20]Notice the slight abuse of notation here. While Ξ^1_{ij} refers to the edge-specific traffic along the primary network, $\Xi^2_{i'j'}$ instead refers to rail flows between i' and j' along the secondary network and is therefore not edge-specific. However, $\Xi^2_{i'j'}$ summarizes railroad traffic in the sense that it refers to any flows between i' and j' no matter their origin or destination on the primary network. This is convenient - as we will argue below - since this is the data moment that is directly observed in the rail traffic data.

$$\widehat{\Xi}_{kl}^{1} = \hat{\chi}^{-\frac{1}{1+\theta\lambda_1}} \left(\hat{\hat{t}}_{kl}\right)^{\frac{\theta}{1+\theta\lambda_1}} \times \left(\hat{y}_k \hat{l}_k^{\beta-1}\right)^{-\frac{\theta}{1+\theta\lambda_1}} \times \left(\hat{l}_l^{\alpha+1} \hat{y}_l^{-\frac{\theta+1}{\theta}}\right)^{-\frac{\theta}{1+\theta\lambda_1}} \tag{40}$$

$$\widehat{\Xi}_{kl}^{2} = \hat{\hat{s}}_{kk'}^{-\frac{\theta}{1+\theta\lambda}} \hat{\hat{s}}_{l'l}^{-\frac{\theta}{1+\theta\lambda}} \hat{P}_k^{-\frac{\theta}{1+\theta\lambda_2}} \hat{\Pi}_l^{-\frac{\theta}{1+\theta\lambda_2}} \hat{\tau}_{k'l'}^{-\theta} \left(\sum_l \frac{\Xi_{k'l'}^{2}}{\sum_{l'} \Xi_{k'l'}^{2}} \hat{\tau}_{k'l'}^{-\theta} \hat{s}_{l'l}^{-\theta} \hat{\Pi}_l^{-\theta}\right)^{-\frac{\theta\lambda}{1+\theta\lambda}} \left(\sum_l \frac{\Xi_{k'l'}^{2}}{\sum_{l'} \Xi_{k'l'}^{2}} \hat{\tau}_{k'l'}^{-\theta} \hat{s}_{l'l}^{-\theta} \hat{P}_l^{-\theta}\right)^{-\frac{\theta\lambda}{1+\theta\lambda}} \tag{41}$$

Corollary 1 allows us to account for the changes in the observed traffic flows. This can be a convenient tool to analyse the implied environmental impact of infrastructure investments.

3.5 Simulated Example

To illustrate the impact of multimodal transportation, we provide a stylized simulation whose results are indicated in Figure A.6. We simulate an economy with 25 locations arranged on 5 x 5 grid. All locations are identical and connected via a grid of roads. Furthermore, we also allow for a secondary transportation system that connects the vertical axes of locations at the center of the system. Panel (a) presents the initial equilibrium. The coloring of the edges indicates the traffic intensity both along the primary and the secondary network with darker colors indicating higher levels of traffic. The coloring of the nodes indicates the population levels going from the colder towards the warmer part of the color spectrum, with higher levels being indicated by warmer colors. In panel (b) we increase the intermodal switching cost. The direct effect is that this decreases the traffic along the vertical axis on the secondary transportation network. It also decreases the attractiveness of locations with access to the secondary transportation network with loocations both on the vertical axis and right next to it decreasing their population levels. There is also modal diversion happening with more traffic flowing along the road routes adjacent to the vertical axis. Overall, welfare is substantially reduced.

4 From Theory to Data

We now quantify our model so that we can employ it to evaluate the welfare impact of multimodal infrastructure improvements. As in Proposition 1, the counterfactual equilibrium crucially depends on the model parameters, in particular the strength of the congestion forces on the primary network and at terminals (λ_1, λ_2). While our calibration broadly follows Allen and Arkolakis (2022) who provide an estimate for the strength of congestion on the primary network, we have introduced a new parameter that pins down congestion at intermodal terminals,

λ_2. In this section we present how the relationship between dwell times at intermodal facilities and throughput can be used to estimate the strength of congestion and the magnitude of this parameter. We pin down this parameter in two steps. First, in Subsection 4.1, we examine ship dwell times and their responsiveness to port traffic. In Subsection 4.2, we then investigate the impact of port traffic on the multimodal transport network, specifically focusing on dwell times at local rail stations. The sum of these elasticities inform the overall strength of congestion at any intermodal facility. Lastly, to motivate the key channel in our model - modal diversion - we illustrate modal diversion in the context of a large infrastructure investment project that lowered transportation costs on railroads between Virginia and Chicago, called the Heartland Corridor.

4.1 Estimation of Port Congestion

In this subsection, we measure port congestion by estimating an elasticity of ship dwell time with respect to port traffic. We estimate the following regression (Column (3), Table 1):

$$\ln \text{Ship Dwell Time}_{spdmy} = \beta_1 \ln \text{Port Traffic}_{pdmy} + \delta_{dmy} + \alpha_{spm} + \epsilon_{spdmy} \tag{42}$$

where Ship Dwell Time$_{spdmy}$ is the number of hours ship s spent at port p on day of the week d month m and year y, Port Traffic$_{pdmy}$ is the 28-day moving average amount of port traffic at port p ending on day d month m and year y, δ_{dmy} is day-month-year fixed effects, and α_{spm} is ship-port-month fixed effects. The key parameter of interest, β_1, captures the elasticity of ship dwell times with respect to port traffic. Standard errors are clustered at the port level.

The day-month-year fixed effects control for aggregate events that affects all ships. The ship-port-month fixed effects control for fixed and time-varying characteristics at the ship-port level. Fixed ship-port characteristics include time-invariant comparative advantage differences for different ports that result in larger ships being received at these ports which mechanically take longer time to unload, for example ports with deep natural harbors. It also includes fixed ship characteristics like size and fixed port characteristics like its geography. Time-varying ship-port characteristics account for potential technology changes over time that ports can undertake that might affect ship dwell times, for example technology upgrades over time to accommodate larger ships.

We find that a one percent increase in port traffic results in a statistically significant increase in ship dwell times by 0.13 percent (Column (3), Table 1). This elasticity is robust to specifications with ship fixed effects and port-month fixed effects separately (Column (1), Table 1) as well as with ship-port fixed effects and port-month fixed effects separately (Column (2), Table 1). In order to account for the extraordinary pandemic period, we include an indicator for the pandemic period (post March 2020) in order to estimate separate elasticities for port congestion. We find that our pre-March 2020 estimate is within one standard error of the baseline results (Column (4), Table 1). As expected, our estimate post-March 2020 is slightly higher in magnitude but not statistically different from the pre-period elasticity. Additionally, the West Coast ports have a history of port strikes and slowdowns. These events necessarily result in longer dwell times for ships that are servicing these ports. We exclude all West Coast ports in a robustness check and find that our estimate, while lower in magnitude, is within one standard error of our baseline estimate.

The baseline results use a 28-day moving average of total daily net tonnage at the port. Using a shorter period of the moving average calculation, we find that the elasticity of port traffic with respect to ship dwell times decreases in magnitude. With a shorter period of moving average calculation, the ship dwell times respond less to changes to the average tonnage at the port. Column (1) reproduces our baseline results using the 28-day moving average from Table 1, Column (2) presents the 21-day moving average, Column (3) presents the 14-day moving average, and Column (4) presents the 7-day moving average.

4.2 Multimodal Impact of Port Congestion

In this subsection, we study the effect of port congestion on the multimodal network. In particular, we focus on how port traffic affects the amount of time a rail car spends at t rail station that is local to that port. We estimate the following regression (Column (2), Table 3):

$$\ln \text{Rail Dwell Time}_{rpwmy} = \beta_2 \ln \text{Port Traffic}_{pwmy} + \gamma_{wmy} + \phi_{rpm} + \epsilon_{rpwmy} \tag{43}$$

where Rail Dwell Time$_{vpt}$ is the average number of hours a rail car spends at a rail station r that is in the vicinity of port p for week w month m and year y, Port Traffic$_{pwmy}$ is the total amount of port traffic at port p for week w month m and year y,[21] γ_{wmy} is week-month-year fixed effects, and ϕ_{rpm} is week-month-year fixed effects. The key parameter of interest, β_2, captures the elasticity of rail dwell times with respect to port traffic. Standard errors are clustered at the port level.

[21]This measure, as mentioned from the previous subsection, is at the daily level. In order to match the rail dwell time dataset, we aggregate it up to the weekly level. We start our week on a Monday since we observe in our data that most ships tend to enter a port on Mondays.

Table. 1. Elasticity of Port Traffic with respect to Ship Dwell Times

	(1)	(2)	(3)	(4)	(5)
Port Traffic	0.129	0.123	0.133		0.111
	(0.0404)	(0.0394)	(0.0401)		(0.0390)
Port Traffic × Before Mar 2020				0.131	
				(0.0421)	
Port Traffic × After Mar 2020				0.137	
				(0.0418)	
Day-Month-Year FE	✓	✓	✓	✓	✓
Ship-Port-Month FE			✓	✓	✓
Port-Month FE	✓	✓			
Ship-Port FE		✓			
Ship FE	✓				
Without West Coast Ports					✓
Observations	59551	59551	59551	59551	44920
R^2	0.65	0.73	0.81	0.81	0.72
F	10.23	9.76	10.94	5.70	8.17

***Notes*:** Robust standard errors in parentheses are clustered by port. All variables are in logs. Port traffic is the 28-day moving average of total daily net tonnage at the port. Weighted by ship net tonnage.

The week-month-year fixed effects control for aggregate events that affects all rail stations. The rail-port-month fixed effects control for fixed and time-varying characteristics at the rail-port level. Fixed rail-port characteristics include time-invariant comparative advantage differences across ports that result in larger capacity trains servicing the rail stations close to these ports which mechanically take longer time to unload. It also includes fixed rail station characteristics and fixed port characteristics that take into account their geography. Time-varying rail-port characteristics account for potential technology changes over time that ports can undertake that might affect rail station dwell times.

We find that a one percent increase in port traffic results in a statistically significant increase in rail dwell times by 0.03 percent (Column (2), Table 3). This elasticity is robust to specifications with rail station fixed effects and port-month fixed effects separately (Column (1), Table 3). As mentioned previously, the West Coast ports have a history of port strikes and slowdowns. These events necessarily result in longer dwell times for ships and these delays can potentially also spillover to the multimodal network and increase dwell times at rail stations. We exclude all West Coast ports in a robustness check and find that our estimate is within one standard error of our baseline estimate (Column (3), Table 3). Additionally, we match local railroads to ports

Table. 2. Elasticity of Port Traffic with respect to Ship Dwell Times

	(1)	(2)	(3)	(4)
Port Traffic	0.133	0.105	0.0911	0.0373
	(0.0401)	(0.0322)	(0.0253)	(0.0177)
Day-Month-Year FE	✓	✓	✓	✓
Ship-Port-Month FE	✓	✓	✓	✓
Moving Average (Days)	28	21	14	7
Observations	59551	59551	59549	59520
R^2	0.81	0.81	0.81	0.81
F	10.94	10.71	12.97	4.45

***Notes*:** Robust standard errors in parentheses are clustered by port. All variables are in logs. Column (1) estimates the elasticity using the 28-day moving average of total daily net tonnage at the port and is replicated from the baseline results in Column (3) Table 1. Column (2) presents the 21-day moving average, Column (3) presents the 14-day moving average, and Column (4) presents the 7-day moving average. Weighted by ship net tonnage.

by extending the port area in our dataset. The buffer area we used in our baseline result is 150km which captures 7 ports and 12 rail stations. As a robustness check, we extend the buffer area around the ports to 200km which captures 8 ports and 14 rail stations. We find that our estimate is within one standard error of our baseline estimate (Column (4), Table 3). The magnitude of this estimate is smaller, due to the impact of port traffic being more muted on rail stations that are further away. Subsequent increases to the buffer area correspondingly result in even smaller estimates. In order to account for the extraordinary pandemic period, we again include an indicator for the pandemic period (post March 2020). We find that our pre-March 2020 is within one standard error of the baseline results (Column (5), Table 3). Again as expected, our post-March 2020 estimate is higher in magnitude.

4.3 Railroad Infrastructure Improvement: Heartland Corridor

In order to illustrate the main channel of our model, i.e. modal diversion, we present preliminary empirical evidence on the impact of a large infrastructure improvement that is mode-specific on US regional rail traffic flows. The Heartland Corridor is a $150 million infrastructure plan to increase capacity of rail lines by increasing height clearance in tunnels/bridges to allow for double-stack intermodal trains. This new route saves double-stack trains 230 miles (up to 2 days) between Norfolk & Chicago (Board, National Academies of Sciences and Medicine, 2017).

Using waybill rail data, we estimate the evolution of rail shipment flows as a result of Heart-

Table. 3. Elasticity of Rail Dwell Times with respect to Port Traffic

	(1)	(2)	(3)	(4)	(5)
Total Port Traffic	0.0267	0.0268	0.0273	0.0245	
	(0.00517)	(0.00518)	(0.00662)	(0.00641)	
Total Port Traffic × Before Mar 2020					0.0258
					(0.00886)
Total Port Traffic × After Mar 2020					0.0305
					(0.0134)
Port Buffer Area	150km	150km	150km	200km	150km
Week-Month-Year FE	✓	✓	✓	✓	✓
Rail Station-Port-Month FE		✓	✓	✓	✓
Port-Month FE	✓				
Rail Station FE	✓				
Without West Coast Ports			✓		
Observations	4087	4087	3361	4813	4087
R^2	0.81	0.81	0.81	0.81	0.81
F	26.79	26.87	17.01	14.65	23.10

Notes: Robust standard errors in parentheses are clustered by port. All variables are in logs. Local railroads are determined by a 150km buffer area around the ports.

land Corridor:

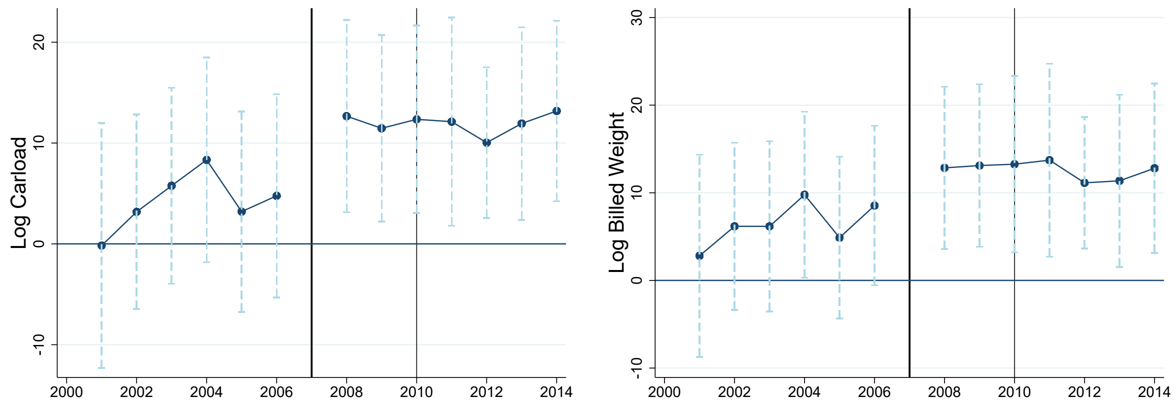
$$Y_{odt} = \sum_{t'=2001\backslash 2007}^{2014} \beta_{t'} \Delta D_{od} 1_{t'} + \lambda_{ot} + \theta_{dt} + \text{Distance}_{od} + \varepsilon_{odt}$$

where Y_{odt} is the trade flows from origin o to destination d at year t, ΔD_{od} is change in log shortest rail distance between o and d before and after the Corridor, indicator $1_{t'}$ equals one for year t', λ_{ot} is origin-year-level fixed effects, θ_{dt} is destination-year fixed effects, and Distance_{od} is the distance between origin and destination. The key parameter of interest, $\beta_{t'}$, is the cumulative Heartland Corridor impact on the trade flows outcome by each year. Corridor construction started in 2007 and is the base year for outcome changes. Preliminary results show that the Heartland Corridor has resulted in an increase of rail traffic immediately in origin-destination pairs that have been impacted more after its implementation, after which they accumulate more slowly but continue to rise (Panel (B), Figure 7).

This large improvement on rail network can potentially have spillover effects onto other modes of transportation. Utilizing the Commodity Flow Surveys (CFS, Bureau of Transportation Statistics), we present suggestive evidence on the modal diversion effects of the Heartland Corridor on road traffic. The CFS is published every 5 years and so we utilize the 2002, 2007, 2012, and 2017 volumes. Due to data limitations on earlier CFS volumes, we present goods movement between US states via road transport.[22] For each state pair, we calculate their shortest rail

[22] The 2002 CFS does not publish goods movement data at the CFS area by transport mode level. Instead this

Figure 7. Impact of Double-Stack Infrastructure Improvement



(a) Freight Carloads (b) Freight Weight

Notes: The Heartland Corridor started in 2007 and was completed in 2010. Robust standard errors both Panels are clustered by origin and destination. Source: Authors' calculations using confidential rail data from the Surface Transportation Board.

distance before and after the Corridor. Figure 8 compares the road shipments between pairs that have a decrease in rail distance due to the Corridor to pairs that do not. Only state pairs that report non-missing road shipment observations for all 4 CFS volumes are included. Panel (A) presents the road shipments measured in value while Panel (B) is measured in tons with both normalized to their 2002 values. We find that for pairs that would potentially benefit from the Heartland Corridor, the amount of shipments transported by road increased about 40% from 2002 (3.1 billion dollars in 2002 to 4.4 billion dollars in 2012, dashed blue line in Panel (A) Figure 8). Comparatively, pairs that do not have a decrease in their rail distance due to the Corridor see their road shipments almost double over this period (4 billion dollars in 2002 to 7.8 billion dollars in 2012, solid blue line in Panel (A) Figure 8). We see similar patterns using the weight measure–almost no increase in weight shipments for pairs that are impacted between 2002 and 2017 compared to an increase of about 25% for pairs that are not (Panel (B) Figure 8).[23]

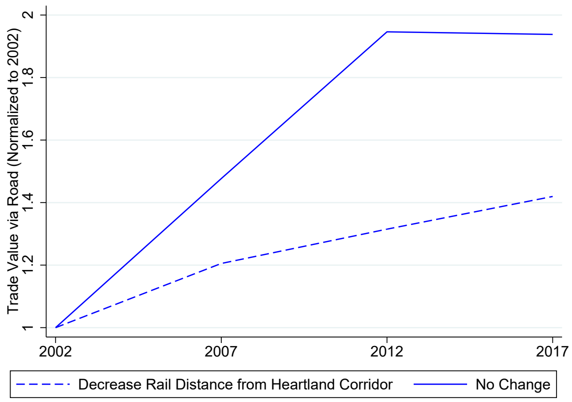
While the CFS also report shipments by multiple modes, this variable has many missing observations particularly in the earlier volumes.[24] We focus on one state pair where we have consistent data for these road and multiple transport shipments—Illinois and Virginia. As mentioned earlier, the Corridor saves double-stack trains 230 miles between Norfolk & Chicago (Board, Na-

information is only available at the state level.

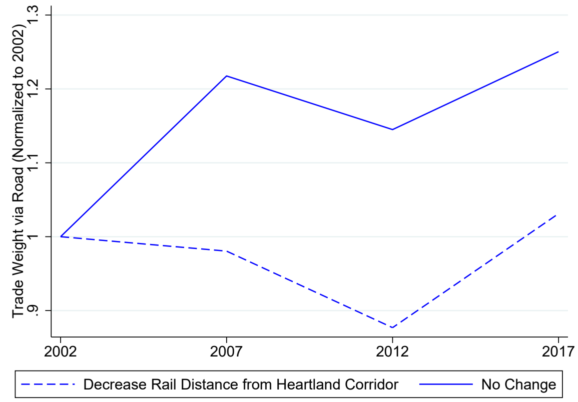
[23]3.1 million tons in 2002 to 3.2 million tons in 2017 for impacted pairs, 8 million tons in 2002 to 10 million tons in 2017 for the unimpacted pairs.

[24]The CFS defines multiple mode shipments as shipments which uses two or more of the following modes of transportation in addition to parcel delivery/courier/US parcel post shipments: truck, railroad, water, pipeline, and air. We acknowledge that this is a noisy measure of multiple-mode shipment since we are unable to disaggregate this measure further due to lack of observation.

Figure 8. Comparison of Road Shipments for Locations Impacted by Double-Stack Rail Improvement



(a) Value



(b) Weight

Notes: The Heartland Corridor started in 2007 and was completed in 2010. Source: Authors' calculations using Commodity Flow Survey (Bureau of Transportation Statistics).

tional Academies of Sciences and Medicine, 2017). This translates into time savings of up to 2 days. Figure 9 plots the share of shipment between road and multiple modes for Illinois and Virginia. For both value and weight, we see an increase in shipment via multiple modes over this time period coupled with a corresponding decrease in road shipment share. These figures present suggestive evidence on the modal diversion effects of a large-scale infrastructure improvement on one transport mode.

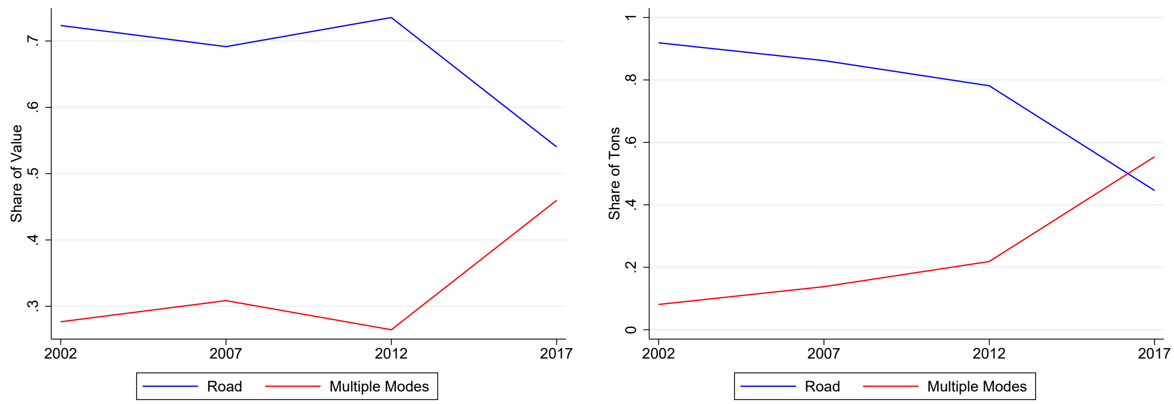
5 Welfare Impact of Infrastructure Investments in a Multimodal Transport Network

We apply our multimodal economic geography framework to evaluate the welfare impact of small improvements in the operation of intermodal terminals, taking into account both the primary and secondary transportation network.

5.1 The Welfare Benefit of Investing in Terminals

While previous papers have focused on estimating the welfare effects of improving individual segments of the US road network, and in particular of the US highway network, less is known about the welfare impact of improving the degree to which the US multimodal transport network is interconnected. In order to evaluate this, we will use the counterfactual equations of Proposition (1) to estimate the aggregate welfare impact ($\hat{W} = \hat{\chi}^{-\frac{1}{\theta}}$) of a small (1%) improvement to the switching cost, $s_{ii'}$, at each intermodal node across the system. This procedure

Figure 9. Road and Multiple-Mode Shipments between Illinois and Virginia



(a) Value

(b) Weight

Notes: The Heartland Corridor started in 2007 and was completed in 2010. Source: Authors' calculations using Commodity Flow Survey (Bureau of Transportation Statistics).

requires three ingredients: (1) As in AA2022, we will need data on road traffic $\{\Xi^1_{kl}\}$ and income $\{Y_i = E_i\}$; (2) data on bilateral railroad traffic $\{\Xi^2_{k'l'}\}$ and (3) knowledge of the model parameters $\{\alpha, \beta, \theta, \lambda_1, \lambda_2\}$. In what follows we describe our data sources, data construction and calibration of key parameters.

5.1.1 Road and Rail Network

We follow AA2022 for the income and road traffic data and refer to their paper for details. In a nutshell, their procedure proceeds in three steps: First, they create a sparse graph representation of the underlying road network by collapsing the high-dimensional geo-spatial information contained in the original shapefiles and only preserving nodes that are either endpoints or intersections. Furthermore, core-based statistical areas (CBSAs) are represented by a singular node along the network. Their resulting graph consists of 228 nodes and and 704 edges. Second, they construct a weighted graph by including traffic data. To do so they obtain the average annual daily traffic (AADT) from the the 2012 Highway Performance Monitoring System (HPMS) dataset by the Federal Highway Administration and allocate it to individual links by constructing a length-weighted average of AADT. Third, they append income and population data to the graph. Income and population is allocated to individual nodes by summing the population and reporting the median income of cities within 25 miles of the node. The raw geo-spatial information is presented in panel (b) in Figure A.2, while panel (b) in Figure A.3 demonstrates the traffic data and panel (b) in Figure A.5 presents the resulting graph. We augment the information regarding the road network by providing additional information on the rail network. We proceed in three steps:

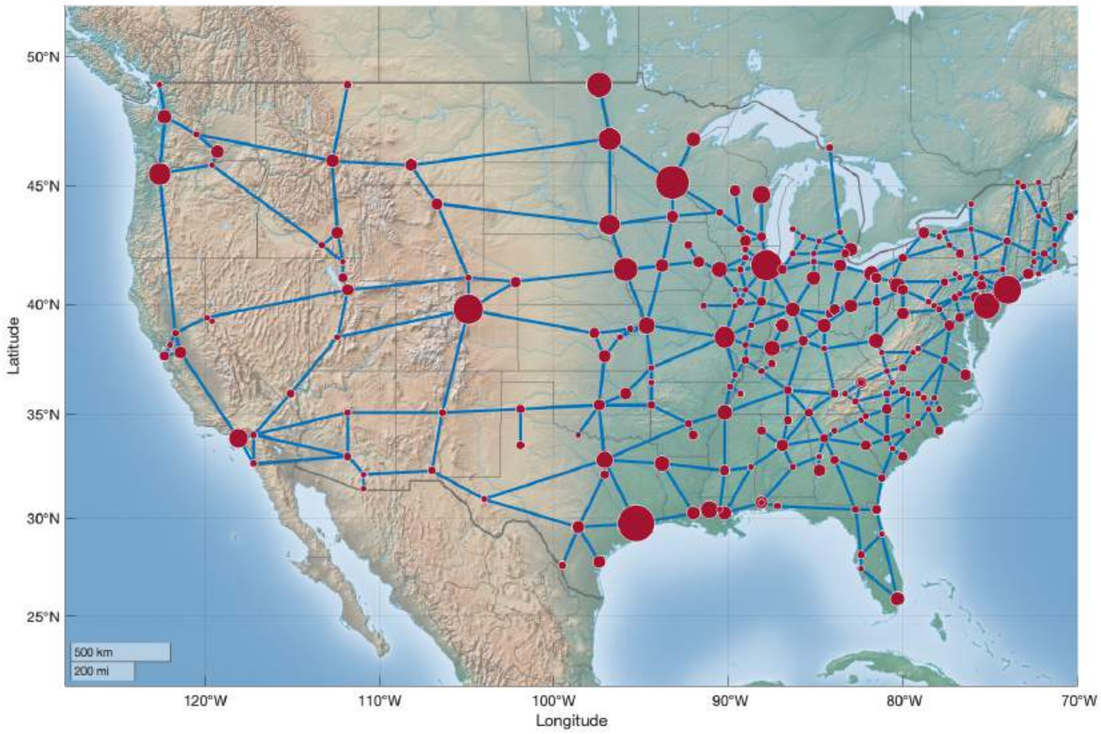
First, we use detailed geo-spatial information from the U.S. Census Bureau's Topologically Integrated Geographic Encoding and Referencing (TIGER) Database to construct a graph representation of the US intermodal rail network. To do so, we subset the original network to those segments that are owned by Class I carriers[25] and are compatible with multimodal transportation. We create a sparse graph representation by collapsing the geo-spatial information and only preserving nodes that are either endpoints or intersections of the network. Crucially, intersections might also represent terminal stations where a transfer from rail-to-road or road-to-rail is possible. In order to isolate those nodes, we include information from the National Transportation Atlas Database (NTAD) maintained by the Department of Transportation (DOT) on the location of intermodal freight facilities. The raw geo-spatial information for the rail network only is presented in panel (a) in Figure A.2, while the combined information is presented in panel (a) in Figure A.5. The resulting graph representation is indicated in panel (b) in the same figure.

Second, we enrich this graph by appending the rail traffic data presented in Section 2.2.3. In a first step, to illustrate the data we subset to intermodal traffic, and impute the shortest routes between origin, interchanges and destination. This allows us to assign total tonnage to individual rail segments along the rail network. The resulting flow map is presented in panel (a) of Figure A.3. For the counterfactual exercise, we need to obtain information on bilateral flows along the secondary network, $\{\Xi^2_{k'l'}\}$. To calculate these flows from the rail traffic data and match them against the railroad network, we match the origin and location railroad station against the nodes in our graph representation of the network and sum total flows for each bilateral origin destination pair.

Third, we append data on container volumes at the top US ports. In order to do so we find the closest node to each port on the primary network. The geo-spatial data is obtained from the US Army Corps of Engineers and the container volumes have been collected from the Port Performance Freight Statistics Program maintained by the Department of Transportation (DOT). The data is visualized in a geographic bubble map in Figure A.4.

[25]Class I railroads are the largest carriers operating on the US railroad system. They were originally in 1992 defined to be those carriers above \$ 250m dollars of revenue, a cutoff that has been adjusted for inflation since. In 2021 the threshold stands at approximately \$943m. There are currently seven class I carriers and they make up the large majority of the domestic rail freight market.

Figure 10. Welfare Gains



Notes: The figure visualizes the welfare impact of lowering the transshipment cost in each node by 1 percent. The larger the dots, the larger the welfare gains. The blue lines indicate the graph representation of the primary road network. State boundaries are included.

5.1.2 Calibration

In a last step before conducting the counterfactual analysis we discuss our choice of the model parameters $\{\alpha, \beta, \theta, \lambda_1, \lambda_2\}$. The calibration of the first four parameters follows AA2022. They in turn follow the literature and in particular the seminal contribution by Ahlfeldt et al. (2015) in setting $\theta = 6.83$, $\alpha = -0.12$ and $\beta = -0.1$. They also provide an estimate for λ_1 by regressing the observed speed on individual highway segments against a measure of instrumented traffic. Their implied value for the primary network congestion parameter is $\lambda_1 = 0.092$.

Finally, we leverage our analysis of port and rail station dwell times to inform our calibration of the strength of congestion at intermodal facility. We follow the transportation literature and conceive of intermodal facilities as a multi-stage process where - in the case of ports - quay operations proceed stack crane operations before truck and rail handling operations transition individual container out of the intermodal facility (Roy, De Koster and Bekker, 2020). Each step in this operation might be subject to congestion that arises due to capacity constraints and queues that might be generated. The total congestion elasticity, λ_2, is therefore given by the

Table. 4. Ranking: Welfare Benefit of Investing in Terminal Operations

	CBSA_Name	originid	Welfare	benefit
1	Houston-Sugar Land-Baytown, TX	142	0.0015	287.10
2	Minneapolis-St. Paul-Bloomington, MN-WI	166	0.0013	241.95
3	Chicago-Joliet-Naperville, IL-IN-WI	248	0.0011	205.63
4	Denver-Aurora-Broomfield, CO	82	0.0010	191.97
5	New York-Northern New Jersey-Long Island, NY-NJ-PA	561	0.0009	174.07
6	Philadelphia-Camden-Wilmington, PA-NJ-DE-MD	548	0.0008	147.61
7	Cavalier, ND	118	0.0007	137.90
8	Omaha-Council Bluffs, NE-IA	131	0.0007	125.16
9	Fargo, ND-MN	126	0.0006	112.79
10	Portland-Vancouver-Hillsboro, OR-WA	17	0.0006	110.13

Notes: The table shows the ten terminals where a one percent reduction of the transportation cost generates the highest benefit. Column (2) indicates the CSBA name of the node. Column (4) indicates the welfare change in percentage points and finally Column (5) indicates how much US GDP would need to increase in order to match the overall welfare gain indicated in the previous column. For an extended version see Table A.1.

composite of the quay side and the rail station congestion. We therefore calibrate the congestion parameter to the sum of the elasticities presented in Section 4.1 and Section 4.2 respectively.[26] We therefore obtain as the total congestion elasticity, $\lambda_2 = \beta_1 + \beta_2 = 0.1363$.

5.1.3 Results

Given the observed road traffic data, railroad traffic, port traffic[27] and calibrated parameters, we calculate the aggregate welfare elasticity to a 1% reduction in iceberg transportation costs at terminal nodes, i.e. $d \ln s_{ii'} = 0.01$. The results are visualized in the map in Figure 10 and the top 10 highest impact nodes are listed in Table 4. Nodes that are highly central to the transportation system have the highest impact and represent bottlenecks on the US system. The implied welfare benefit of alleviating congestion or equivalently lowering transportation cost in some of the most central nodes could represent a welfare gain equivalent to increasing US GDP between 200-300 million USD (in 2012 USD).

[26] While we obtain these elasticities in the context of port operations, conceptually inland intermodal facilities operate symmetrically and face similar multi-stage processes with sequential queuing dynamics.

[27] In Online Appendix D.1 we show how the equilibrium conditions can be extended to allow for international imports and exports at coastal ports.

6 Conclusion

The movement of goods from origin to destination involves multiple modes of transportation, including highways, railroads, oceans, and waterways. Correspondingly, intermodal terminals play an important role in facilitating how goods are transported over this network. We study multimodal transport networks and their impact on the economic and environmental returns to new technology and infrastructure investments. In particular, we focus on how these outcomes will depend on the geography of the multimodal transportation network, the placement of intermodal terminals that allow for switches between modes of transportation, as well as the relative cost of transportation across modes. By incorporating these features we provide a framework that allows us to realistically evaluate infrastructure policies taking the complete domestic transportation network into account.

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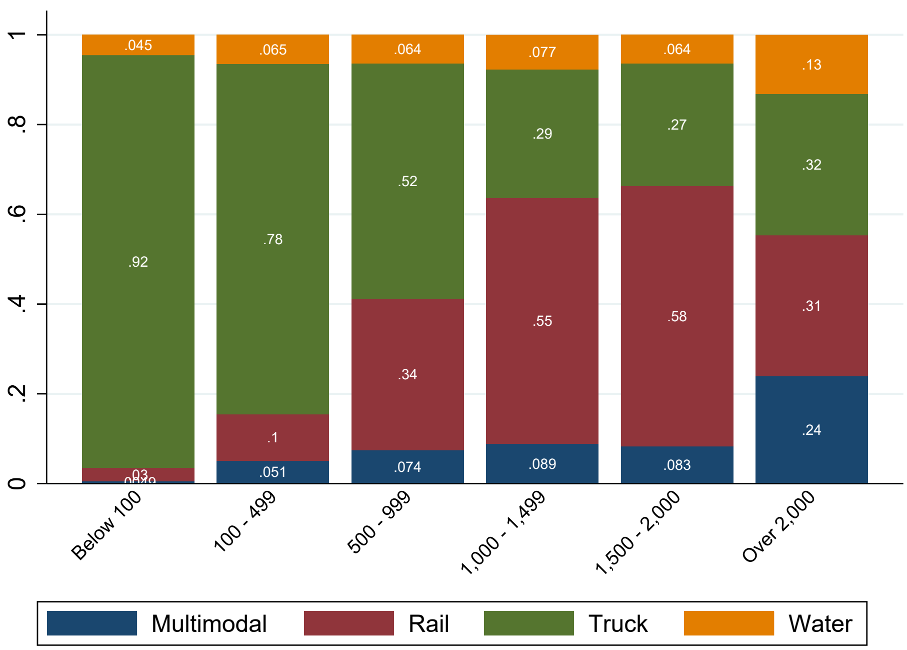
Tables and Figures

Figure A.1. US Transport Mode Weight Shares by Distance, 2018



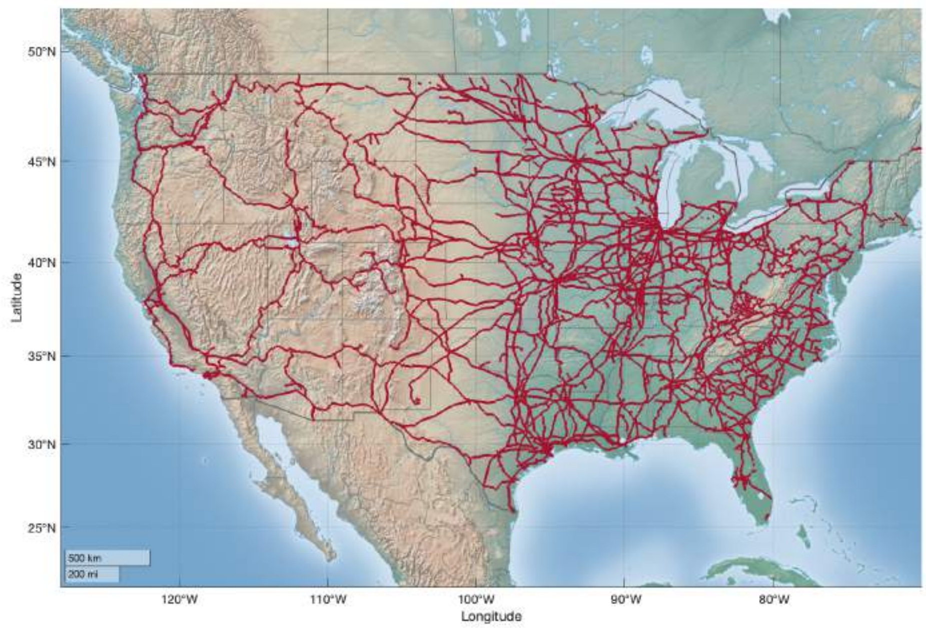
Notes: This figure plots the observed weight share of cargo transported by different modes across various distances. Multimodal indicates cargo movement that involves more than one mode. Source: Freight Analysis Framework, US Department of Transportation, and authors' calculations.

Table. A.1. Ranking: Welfare Benefit of Investing in Terminal Operations

	CBSA_Name	originid	Welfare	benefit
1	Houston-Sugar Land-Baytown, TX	142	0.0015	287.10
2	Minneapolis-St. Paul-Bloomington, MN-WI	166	0.0013	241.95
3	Chicago-Joliet-Naperville, IL-IN-WI	248	0.0011	205.63
4	Denver-Aurora-Broomfield, CO	82	0.0010	191.97
5	New York-Northern New Jersey-Long Island, NY-NJ-PA	561	0.0009	174.07
6	Philadelphia-Camden-Wilmington, PA-NJ-DE-MD	548	0.0008	147.61
7	Cavalier, ND	118	0.0007	137.90
8	Omaha-Council Bluffs, NE-IA	131	0.0007	125.16
9	Fargo, ND-MN	126	0.0006	112.79
10	Portland-Vancouver-Hillsboro, OR-WA	17	0.0006	110.13
11	St. Louis, MO-IL	193	0.0005	99.98
12	Sioux Falls, SD	123	0.0005	93.27
13	Los Angeles-Long Beach-Santa Ana, CA	21	0.0004	82.09
14	Green Bay, WI	246	0.0004	74.64
15	Baton Rouge, LA	196	0.0004	66.74
16	Dallas-Fort Worth-Arlington, TX	102	0.0003	64.23
17	Kansas City, MO-KS	148	0.0003	63.54
18	Davenport-Moline-Rock Island, IA-IL	191	0.0003	57.06
19	Evansville, IN-KY	276	0.0003	54.21
20	Memphis, TN-MS-AR	213	0.0003	54.05
21	Cleveland-Elyria-Mentor, OH	382	0.0003	53.87
22	New Castle, PA	419	0.0003	53.52
23	Shreveport-Bossier City, LA	168	0.0003	52.57
24	Charleston, WV	402	0.0003	50.05
25	Indianapolis-Carmel, IN	288	0.0002	47.01
26	Duluth, MN-WI	179	0.0002	46.65
27	Fort Wayne, IN	309	0.0002	46.32
28	Seattle-Tacoma-Bellevue, WA	40	0.0002	46.14
29	Miami-Fort Lauderdale-Pompano Beach, FL	497	0.0002	44.64
30	Cincinnati-Middletown, OH-KY-IN	322	0.0002	43.98

Notes: The table shows the thirty terminals where a one percent reduction of the transportation cost generates the highest benefit. Column (2) indicates the CSBA name of the node. Column (4) indicates the welfare change in percentage points and finally Column (5) indicates how much US GDP would need to increase in order to match the overall welfare gain indicated in the previous column.

Figure A.2. US Domestic Freight Transport System

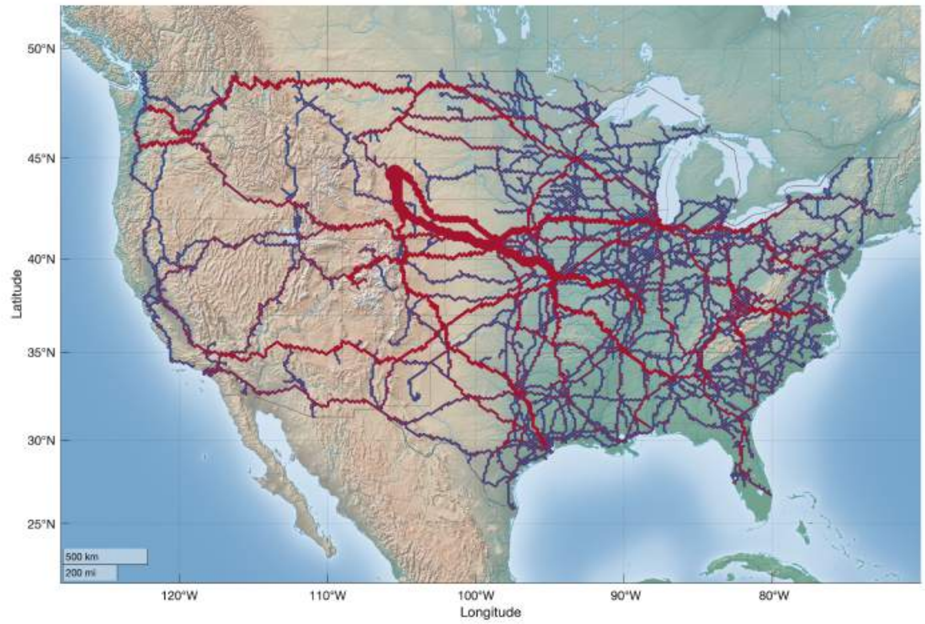


(a) Class I Multimodal Railroad Network

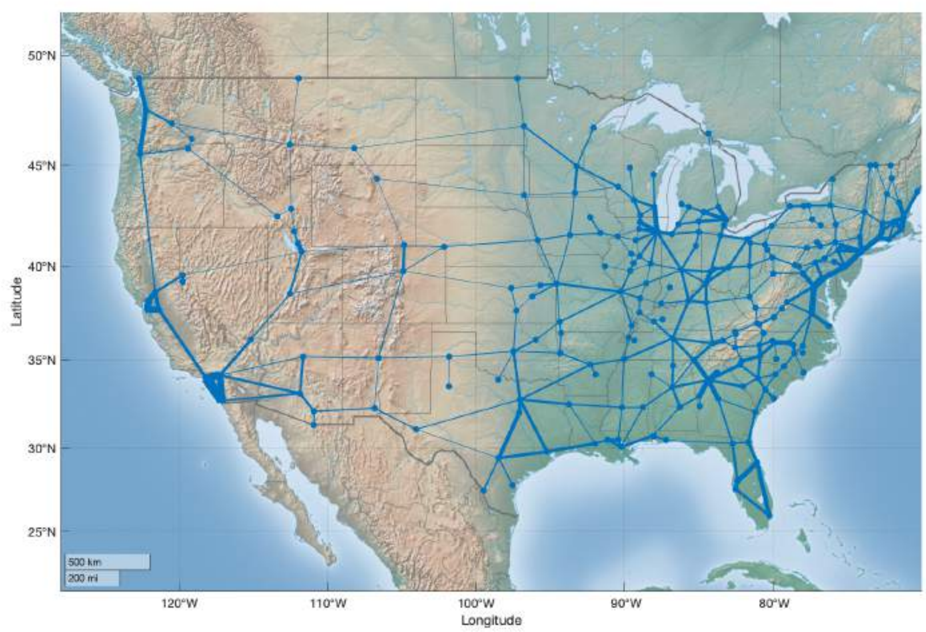
(b) Interstate Highway System (IHS)

Notes: Panel (a) shows Class I railroad network in the US. We obtain the original GIS information from the U.S. Census Bureau's Topologically Integrated Geographic Encoding and Referencing (TIGER) Database. We subset to the segments that are owned by Class I carriers and are compatible with multimodal transport. Panel (b) shows the interstate highway system (IHS).

Figure A.3. Traffic along the US Domestic Freight Transport System



(a) Traffic along the Class I Railroad Network



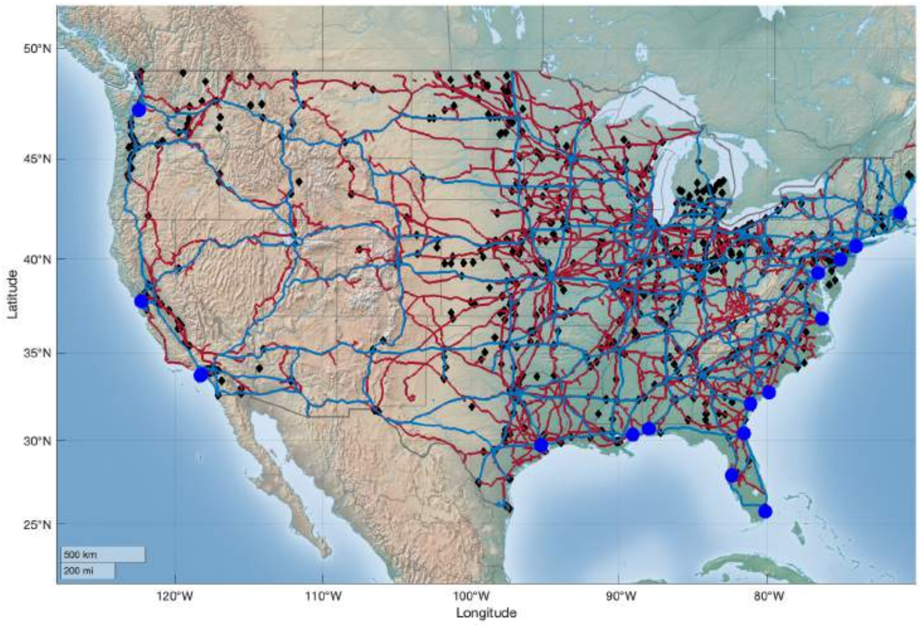
(b) Traffic along the Interstate Highway System (IHS)

Notes: Panel (a) shows the routed traffic along the multimodal Class I railroad network. We use the rail traffic data described in 2.2.3, subset to intermodal traffic, and impute shortest routes between origin, interchanges and destination to assign total tonnage to individual rail segments along the multimodal network. Panel (b) presents the traffic along the graph representation of the interstate highway system, depicting data from the 2012 Highway Performance Monitoring System (HPMS) dataset by the Federal Highway Administration.

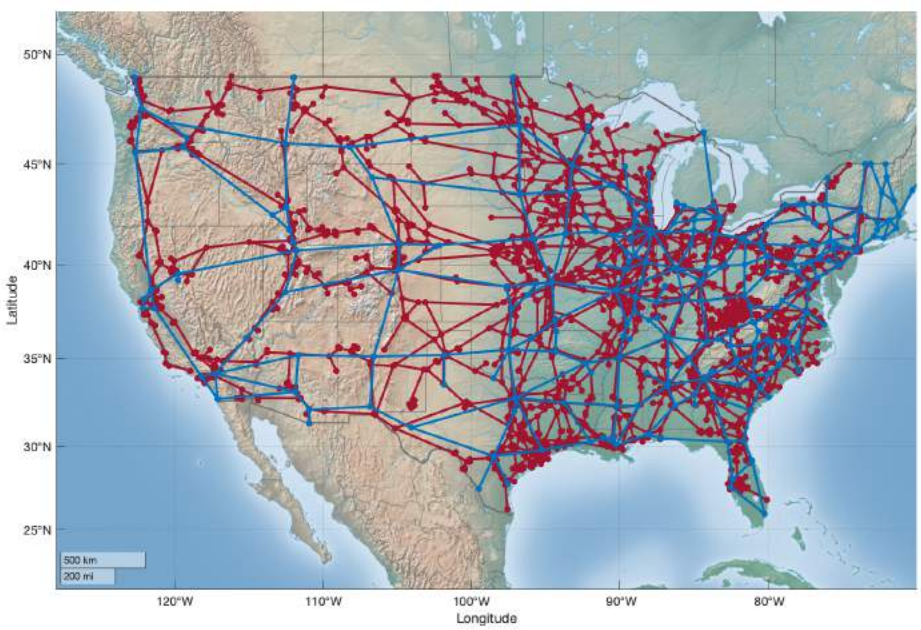
Figure A.4. International Trade at US Ports

Notes: The figure presents a geographic bubble chart where ports are represented by a bubble and the size of the bubble is proportional to the total twenty foot equivalent units (TEUs) that are being handled by each port. Source: US Army Corps of Engineers, Port Performance Freight Statistics Program

Figure A.5. Graph Representation for US Freight Network



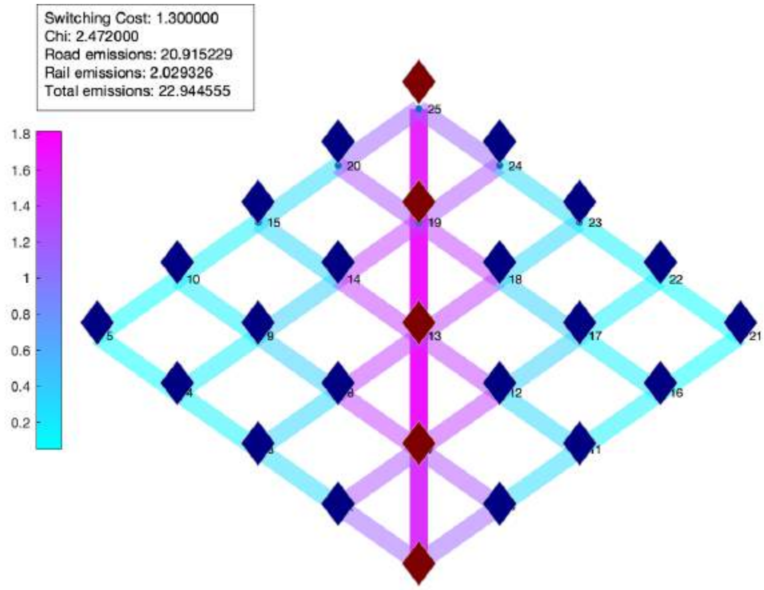
(a) Railroads, Highways and Freight Terminals



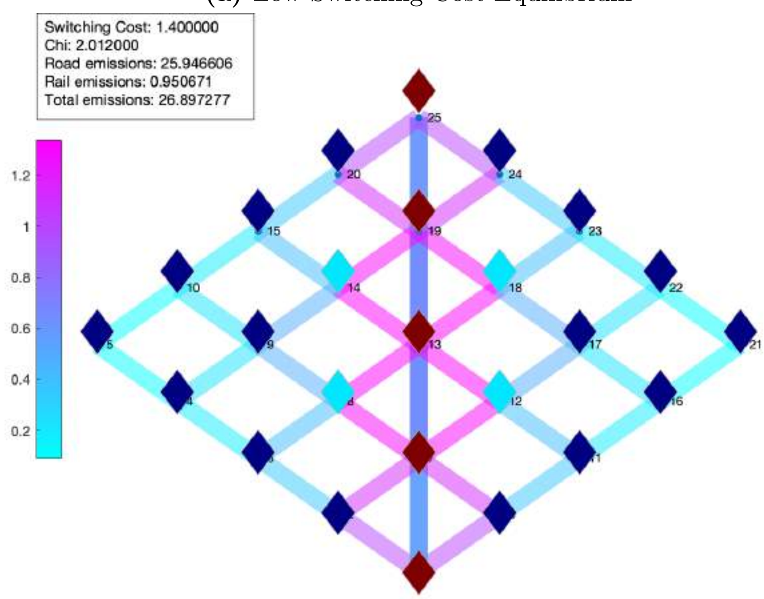
(b) Rail and Road Undirected Graph

Notes: Panel (a) shows the combined US multimodal freight network. We obtain the original GIS information from the U.S. Census Bureau's Topologically Integrated Geographic Encoding and Referencing (TIGER) Database. The red lines indicate the Class I multimodal railroad network. The blue lines indicate the interstate highway system (IHS). Black diamonds indicate freight terminals that are owned by Class I operators and allow for road-to-rail or rail-to-road intermodal movements. The blue circles indicate the top 18 ports. Panel (b) shows the graph representation of the road (blue) and rail (red) network. Nodes are either population centers or intersections. Details for the construction of the graph are given in Section 5.1.1.

Figure A.6. Simulation: A System of Cities with Multimodal Transport



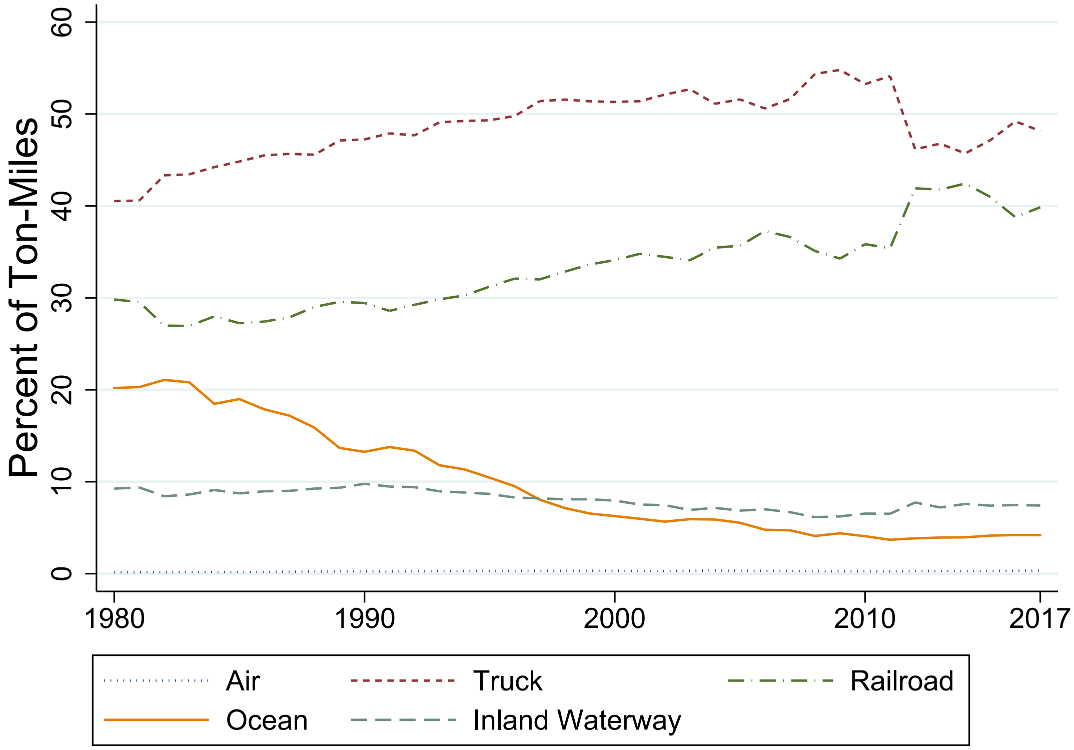
(a) Low Switching Cost Equilibrium



(b) High Switching Cost Equilibrium

Notes: The figures show a simulated economy with 25 cities arranged on a 5 x 5 grid. The cities are connected via a primary road network on the grid and a secondary rail network on the vertical axis.

Figure A.7. US Modal Freight Shares



Notes: This figure shows the US modal freight shares going back to 1980. Source: Bureau of Transportation Statistics

A Appendix: Derivations

This appendix presents derivations for the results in Section 3. Additional derivations are presented in Online Appendix D.1.

A.1 Section 3.1.2: Route, Mode and Trade Shares

Conditional on the consumer knowing the expected transportation cost across all modes, the consumer faces a distribution of prices.

$$p_{ij,rm}(\nu) = \frac{w_i \tau_{ij,r}}{\varepsilon_{ij,r}(\nu)}$$

$\varepsilon_{ij,rm}(\nu)$ is a random variable drawn from a Frechet distribution with cumulative distribution given by

$$F_{ijrm}(\epsilon) = e^{-T_{ijr}\epsilon^{-\theta}}$$

Location i presents location $n$with a distribution of prices,

$$\begin{aligned} G_{ijr}(p) = \Pr\left(P_{ijr} \leq p\right) &= 1 - F_{ijr}\left(\frac{w_i \tau_{ij,r}}{p}\right) \\ &= 1 - e^{-T_{ijr}(w_i \tau_{ij,r})^{-\theta} p^{\theta}} \end{aligned}$$

Lowest price will be less than p, unless each source's price is greater than p. So $G_{ijr}(p) = \Pr\left(P_{ijr} \leq p\right)$ is,

$$\begin{aligned} G_{ijrm}(p) &= 1 - \prod_{i,r}\left(1 - G_{ijr}(p)\right) \\ &= 1 - \prod_{i,r} e^{-T_{ijr}(w_i \tau_{ij,r})^{-\theta} p^{\theta}} \\ &= 1 - e^{-\Phi_{ijr} p^{\theta}} \end{aligned}$$

where,

$$\Phi_{ijr} = \sum_{ir} T_{ir}\left(w_i \tau_{ij,r}\right)^{-\theta}$$

If $p_{ni}\left(j\right) = p$ then the probability that $ijrm$ is the lowest cost supplier is:

$$\begin{aligned} \prod_{i' \neq i, r' \neq r} \Pr\left[P_{ijrm} \geq p\right] &= \prod_{i' \neq i, r' \neq r} \left[1 - G_{i'jr'}\right] \\ &= \prod_{i' \neq i, r' \neq r} e^{-T_{ijr}(w_i \tau_{ij,r})^{-\theta} p^{\theta}} \\ &= e^{-\left(\sum_{i,r} T_{ijr}(w_i \tau_{ij,r})^{-\theta}\right) p^{\theta}} \end{aligned}$$

Probability that country i and route r provides a good at the lowest price in country n is,

$$\begin{aligned} \pi_{ijr} &= \int_0^{\infty} \prod_{i' \neq i, r' \neq r} \left[1 - G_{i'jr'}\right] dG_{ijr}(p) \\ &= \int_0^{\infty} \prod_{i' \neq i, r' \neq r} e^{-T_{ijr}(w_i \tau_{ij,r})^{-\theta} p^{\theta}} dG_{ijr}(p) \end{aligned}$$

Replacing with $dG_{ijr}(p) = \left[T_{ijr}\left(w_i\tau_{ij,r}\right)^{-\theta}\theta p^{\theta-1}\right]e^{-T_{ijr}(w_i\tau_{ij,r})^{-\theta}p^{\theta}}dp$

$$\begin{aligned}
\pi_{ijr} &= \int_0^\infty \prod_{i'\neq i,r'\neq r} e^{-T_{ijr}(w_i\tau_{ij,r})^{-\theta}p^{\theta}}dG_{ijr}(p) \\
&= \int_0^\infty \prod_{i'\neq i,r'\neq r} e^{-T_{ijr}(w_i\tau_{ij,r})^{-\theta}p^{\theta}}\left[T_{ijr}\left(w_i\tau_{ij,r}\right)^{-\theta}\theta p^{\theta-1}\right]e^{-T_{ijr}(w_i\tau_{ij,r})^{-\theta}p^{\theta}}dp \\
&= T_{ijr}\left(w_i\tau_{ij,r}\right)^{-\theta}\int_0^\infty \prod_{i,r} e^{-T_{ijr}(w_i\tau_{ij,r})^{-\theta}p^{\theta}}\left[\theta p^{\theta-1}\right]dp \\
&= T_{ijr}\left(w_i\tau_{ij,r}\right)^{-\theta}\int_0^\infty e^{-\left(\sum_{i,r}T_{ijr}(w_i\tau_{ij,r})^{-\theta}\right)p^{\theta}}\left[\theta p^{\theta-1}\right]dp \\
&= T_{ijr}\left(w_i\tau_{ij,r}\right)^{-\theta}\int_0^\infty e^{-\Phi_{ijr}p^{\theta}}\left[\theta p^{\theta-1}\right]dp \\
&= T_{ijr}\left(w_i\tau_{ij,r}\right)^{-\theta}\left[\frac{1}{\Phi_{ijr}}e^{-\Phi_{ijr}p^{\theta}}\right]_0^\infty \\
&= \frac{T_{ijr}\left(w_i\tau_{ij,r}\right)^{-\theta}}{\Phi_{ijr}}
\end{aligned}$$

Replacing with $\Phi_{ijr} = \sum_{ir}T_{ir}\left(w_i\tau_{ij,r}\right)^{-\theta}$, we obtain,

$$\pi_{ijr} = \frac{T_{ijr}\left(w_i\tau_{ij,r}\right)^{-\theta}}{\sum_{ijr}T_{ijr}\left(w_i\tau_{ij,r}\right)^{-\theta}}$$

replacing with $\tau_{ij,r}^{-\theta} = \left(\prod_{l=1}^{K}t_{r_{t-1},r_l}^{-\theta}\right)$, defining $T_{ijr} \equiv \left(\frac{1}{A_i}\right)^{\theta}$, and distinguishing between unimodal and multimodal routes,

$$\pi_{ij,r} = \frac{\left(w_i/A_i\right)^{-\theta}\left(\prod_{l=1}^{K}t_{r_{t-1},r_l}^{-\theta}\right)}{\sum_{k\in\mathcal{N}}\left(w_k/A_k\right)^{-\theta}\sum_{r'\in\mathcal{R}_{kj}^{1}\cup\mathcal{R}_{kj}^{1,2}}\prod_{l=1}^{K}t_{r_{i-1}',r_l'}^{-\theta}}.$$

as stated above.

A.2 Section 3.3.1: Multimodal Routing and Transportation Cost

Define the $(N_1+N_2)\times(N_1+N_2)$ matrix $\mathbf{A} = \left[a_{ij}\equiv t_{ij}^{-\theta}\right]$. Notice that this adjacency matrix forms a block partitioned matrix, i.e.

$$\mathbf{A} = \begin{bmatrix} \mathbf{A_1} & \mathbf{S} \\ \mathbf{S}' & \mathbf{A_2} \end{bmatrix}$$

where $\mathbf{A_1} = [a_{ij}] = \left[t_{ij}^{-\theta}\right]$ is the adjacency matrix for the primary transportation network, $\mathbf{A_2} = \left[a_{i'j'}\right] = \left[t_{i'j'}^{-\theta}\right]$ is the adjacency matrix for the secondary transportation network, and $\mathbf{S} = \left[s_{ii'}^{-\theta}\right]$ is the diagonal matrix that represents linkages between the primary and secondary transportation network. We can write τ_{ij} from equation (4) by explicitly summing across all possible routes of all possible lengths. To do so, we sum across all locations that are traveled through all the possible paths as follows:

$$\tau_{ij}^{-\theta} = \sum_{K=0}^{\infty}\left(\sum_{k_1=1}^{(N_1+N_2)}\sum_{k_2=1}^{(N_1+N_2)}\cdots\sum_{k_{K-1}=1}^{(N_1+N_2)}a_{i,k_1}\times a_{k_1,k_2}\times\ldots\times a_{k_{K-2},k_{K-1}}\times a_{k_{K-1}j}\right)$$

explicitly recognizing that this sum across all locations through all possible paths can be partitioned into unimodal paths on each transportation network and an arbitrary number of switches between transportation modes, we have,

$$\tau_{ij}^{-\theta} = \sum_{t_1=1}^{N}\sum_{t_2=1}^{N}\dots\sum_{t_S=1}^{N}\left(\left(\sum_{K=0}^{\infty}\mathbf{A}_{1,it_1}^{K}\right)\times s_{t_1t_1'}^{-\theta}\times\dots\times s_{t_s't_s}^{-\theta}\left(\sum_{K=0}^{\infty}\mathbf{A}_{1,T_Sj}^{K}\right)\right)$$

which in matrix notation can be written as,

$$\tau_{ij}^{-\theta} = \sum_{K=0}^{\infty}\left(\left(\sum_{K=0}^{\infty}\mathbf{A}_{1}^{K}\right)\left(\mathbf{S}\left(\sum_{K=0}^{\infty}\mathbf{A}_{2}^{K}\right)\mathbf{S}'\right)\right)^{K}\left(\sum_{K=0}^{\infty}\mathbf{A}_{1}^{K}\right)$$

To simplify this expression let us first define the Leontief inverse for each infrastructure matrix separately, i.e.

$$\sum_{K=0}^{\infty}\mathbf{A}_{1}^{K} = (\mathbf{I}-\mathbf{A}_1)^{-1} \equiv \mathbf{B}$$

$$\sum_{K=0}^{\infty}\mathbf{A}_{2}^{K} = (\mathbf{I}-\mathbf{A}_2)^{-1} \equiv \mathbf{C}$$

We also define - for convenience - the sandwich matrix that adjusts the transport cost along the secondary transportation network for switching costs and therefore traces out the option value of having access to the secondary transportation network,

$$\mathbf{S}\left(\sum_{K=0}^{\infty}\mathbf{A}_{2}^{K}\right)\mathbf{S}' \equiv \mathbf{D}$$

From matrix calculus we can restate the following result that relates the inverse of the Schur complement of the partitioned infrastructure matrix to the geometric sum of matrix operations, specifically,

$$\sum_{K=0}^{\infty}\left(\mathbf{B}^{-1}\mathbf{D}\right)^{K}\mathbf{B}^{-1} = (\mathbf{B}-\mathbf{D})^{-1} \equiv \mathbf{E}$$

applying this result we can write,

$$\begin{aligned}\tau_{ij}^{-\theta} &= \sum_{K=0}^{\infty}\left((\mathbf{I}-\mathbf{A}_1)^{-1}\left(\mathbf{S}(\mathbf{I}-\mathbf{A}_2)^{-1}\mathbf{S}'\right)\right)^{K}(\mathbf{I}-\mathbf{A}_1)^{-1}\\ &= \left[(\mathbf{I}-\mathbf{A_1})-\mathbf{S}\left(\mathbf{I}-\mathbf{A_2}\right)^{-1}\mathbf{S}'\right]_{ij}^{-1}\end{aligned}$$

therefore we can write,

$$\tau_{ij} = e_{ij}^{-\frac{1}{\theta}}$$

Furthermore, the Woodbury matrix identity (see e.g. Horn and Johnson (2012)) states,

$$(\mathbf{A}+\mathbf{UCV})^{-1} = \mathbf{A}^{-1}-\mathbf{A}^{-1}\mathbf{U}\left(\mathbf{C}^{-1}+\mathbf{VA}^{-1}\mathbf{U}\right)^{-1}\mathbf{VA}^{-1}$$

which implies

$$\begin{aligned}
\tau_{ij}^{-\theta} &= \left[(\mathbf{I}-\mathbf{A_1}) - \mathbf{S}\left(\mathbf{I}-\mathbf{A_2}\right)^{-1}\mathbf{S}'\right]_{ij}^{-1} \\
&= \left[\mathbf{B} + \mathbf{BS}\left(\mathbf{A}/\mathbf{A_1}\right)^{-1}\mathbf{S}'\mathbf{B}\right]_{ij} \\
&= \left[(\mathbf{I}-\mathbf{A_1})^{-1} + (\mathbf{I}-\mathbf{A_1})^{-1}\mathbf{S}\left(\mathbf{A}/\mathbf{A_1}\right)^{-1}\mathbf{S}'\left(\mathbf{I}-\mathbf{A_1}\right)^{-1}\right]_{ij}
\end{aligned}$$

where $\mathbf{A}/\mathbf{A_1} := (\mathbf{I}-\mathbf{A_1})^{-1} - \mathbf{S}\left(\mathbf{A}/\mathbf{A_1}\right)^{-1}\mathbf{S}'$ defines the Schur complement of the adjacency matrix A. The expressions corresponds to the expression given in the main text and intuitively decomposes the transport cost into a component that originates from the unimodal paths and another component that originates from the multimodal paths. This result can also directly be obtained by applying to the partitioned matrix A the formula for the inverse of block-partitioned matrices (see e.g. Horn and Johnson (2012)).

A.3 Section 3.3.2: Modal Traffic Flows

We characterize equilibrium traffic at different nodes of the transportation network. First, we reiterate the characterization in AA2022 for the primary network in A.3.1. Second, we characterize traffic between origin and destination nodes on the secondary transportation network in A.3.2. Finally, we characterize traffic at terminal stations taking congestion at the terminal into account in A.3.3.

A.3.1 Traffic on the Primary Network

To begin, we charactertize the expected number of times in which link (k,l) is used in trade between (i,j), π_{ij}^{kl}, which we refer to as link intensity. Summing across all routes from i to j the product of the probability a particular route is used and the number of times that route passes through link (k,l), n_r^{kl} (as some routes may use a link more than once:

$$\pi_{ij}^{kl} \equiv \sum_{r\in\mathcal{R}_{ij}^{1}\cup\mathcal{R}_{ij}^{1,2}} \left(\frac{\pi_{ij,r}}{\sum_{r'\in\mathcal{R}_{ij}^{1}\cup\mathcal{R}_{ij}^{1,2}}\pi_{ij,r'}}\right) n_r^{kl}$$

Beginning with this equation,

$$\begin{aligned}
\pi_{ij}^{kl} &= \sum_{r\in\mathcal{R}_{ij}^{1}\cup\mathcal{R}_{ij}^{1,2}} \frac{\pi_{ij,r}}{\sum_{r'\in\mathcal{R}_{ij}^{1}\cup\mathcal{R}_{ij}^{1,2}}\pi_{ij,r'}} n_r^{kl} \iff \\
\pi_{ij}^{kl} &= \sum_{r\in\mathcal{R}_{ij}^{1}\cup\mathcal{R}_{ij}^{1,2}} \frac{\left(\prod_{l=1}^{K} t_{r_{l-1},r_l}^{-\theta}\right)}{\sum_{r\in\mathcal{R}_{ij}}\left(\prod_{l=1}^{K} t_{r_{l-1},r_l}^{-\theta}\right)} n_r^{kI} \iff \\
\pi_{ij}^{kl} &= \tau_{ij}^{\theta} \sum_{r\in\mathcal{R}_{ij}^{1}\cup\mathcal{R}_{ij}^{1,2}} \left(\prod_{l=1}^{K} t_{r_{l-1},r_l}^{-\theta}\right) n_{r,}^{kl}
\end{aligned}$$

For each route in $r\in\mathcal{R}_{ij}^{1}\cup\mathcal{R}_{ij}^{1,2}$, the value $\prod_{l=1}^{K} t_{rl-1,ni}^{-\theta} n_{r,}^{kl}$ is the transportation costs incurred along the route multiplied by the number of times the routes traverses link$\{k,l\}$. To calculate this, we proceed by summing across all possible traverses that occur on all routes from i to j. To do so, note for any $r\in\mathcal{R}_{ij}^{1}\cup\mathcal{R}_{ij}^{1,2}$ of length K (which we denote as $\mathcal{R}_{ij,K}^{1}$ or $\mathcal{R}_{ij,K}^{1,2}$ for unimodal or multimodal routes of length K respectively), a traverse is possible at any point $B\in[1,2,\ldots,K-1]$ in the route.

Defining $\mathbf{A} \equiv [a_{kl}] = \left[t_{kl}^{-\theta}\right]$ and $\mathbf{B} \equiv [b_{ij}] = \left[\tau_{ij}^{-\theta}\right]$ as above, we can write:

$$\pi_{ij}^{kl} = \frac{1}{b_{ij}} \sum_{K=0}^{\infty} \sum_{B=0}^{K-1} \left(\sum_{r \in \mathcal{R}_{ik,B}^{1} \cup \mathcal{R}_{ik,B}^{1,2}} \prod_{n=1}^{B} a_{r_{n-1},r_n} \right) \times a_{kl} \times \left(\sum_{r \in \mathcal{R}_{lj,B}^{1} \cup \mathcal{R}_{lj,B}^{1,2}} \prod_{n=1}^{K-B-1} a_{r_{n-1},r_n} \right)$$

This can in turn allow us to explicitly anumerate all possible paths from i to $k$of length B and all paths from l to j of length $K-B-1$:

$$\pi_{ij}^{kl} = \frac{1}{b_{ij}} \sum_{K=0}^{\infty} \sum_{B=0}^{K-1} \left(\sum_{n_1=1}^{(N_1+N_2)} \cdots \sum_{n_{B-1}=1}^{(N_1+N_2)} a_{i,n_1} \times \ldots \times a_{n_{B-1},k} \right) \times a_{kl} \times \left(\sum_{n_1=1}^{(N_1+N_2)} \cdots \sum_{n_{K-B-1}=1}^{(N_1+N_2)} a_{l,n_1} \times \ldots \times a_{n_{K-B-1},j} \right),$$

which can be exporessed more succinctly as elements of matrix powers of A:

$$\pi_{ij}^{kl} = \frac{1}{b_{ij}} \sum_{K=0}^{\infty} \sum_{B=0}^{K-1} A_{ik}^{B} \times a_{kl} \times A_{lj}^{K-B-1}$$

A result from matrix calculus is for any $N \times N$ matrix C we have:

$$\sum_{K=0}^{\infty} \sum_{B=0}^{K-1} \mathbf{A}^{B} \mathbf{C} \mathbf{A}^{K-B-1} = \sum_{K=0}^{\infty} \left(\sum_{B=0}^{K-1} \mathbf{A}^{B} \mathbf{C} \mathbf{A}^{K-B-1} \right) = (\mathbf{I} - \mathbf{A})^{-1} \mathbf{C} (\mathbf{I} - \mathbf{A})^{-1}$$

Define C to be an $N \times N$ matrix that takes the value a_{kl} at row k and column l and zeros everywhere else. We obtain:

$$\pi_{ij}^{kl} = \frac{b_{ik} a_{kl} b_{ij}}{b_{ij}} \iff$$
$$\pi_{ij}^{kl} = \frac{\tau_{ik}^{-\theta} t_{kl}^{-\theta} \tau_{lj}^{-\theta}}{\tau_{ij}^{-\theta}}$$

We now derive the gravity equations for traffic over a link. For trade, we sum over all trade between all origins and desitnations, and all routes taken by that trade, to get:

$$\Xi_{kl} = \sum_{i \in \mathcal{N}} \sum_{j \in \mathcal{N}} \sum_{r \in \mathcal{R}_{kj}^{1} \cup \mathcal{R}_{kj}^{1,2}} \pi_{ij,r} n_r^{kl} E_j \iff$$
$$\Xi_{kl} = \sum_{i \in \mathcal{N}} \sum_{j \in \mathcal{N}} \pi_{ij}^{kl} X_{ij} \iff$$
$$\Xi_{kl} = \sum_{i \in \mathcal{N}} \sum_{j \in \mathcal{N}} \frac{\tau_{ik}^{-\theta} t_{kl}^{-\theta} \tau_{lj}^{-\theta}}{\tau_{ij}^{-\theta}} \times \tau_{ij}^{-\theta} \frac{Y_i}{\Pi_i^{-\theta}} \frac{E_j}{P_j^{-\theta}} \iff$$
$$\Xi_{kl} = t_{kl}^{-\theta} \sum_{i \in \mathcal{N}} \tau_{ik}^{-\theta} \frac{Y_i}{\Pi_i^{-\theta}} \sum_{j \in \mathcal{N}} \tau_{lj}^{-\theta} \frac{E_j}{P_j^{-\theta}},$$

Recalling the definition of the consumer and producer market access terms,

$$\Pi_i \equiv \left(\sum_{j=1}^{N} \tau_{ij}^{-\theta} E_j P_j^{\theta} \right)^{-\frac{1}{\theta}} = A_i L_i Y_i^{-\frac{\theta+1}{\sigma}}$$

$$P_j = \left(\sum_{i=1}^{N} \tau_{ij}^{-\theta} Y_i \Pi_i^{\theta} \right)^{-\frac{1}{\theta}}$$

we get,

$$\Xi_{kl} = t_{kl}^{-\theta} \times P_k^{-\theta} \times \Pi_l^{-\theta},$$

Combining with the functional form for congestion,

$$\begin{aligned}
t_{ij} &= \bar{t}_{ij} \left[\Xi_{ij}\right]^{\lambda_1} \iff \\
t_{ij} &= \bar{t}_{ij} \left[t_{ij}^{-\theta} \times P_i^{-\theta} \times \Pi_j^{-\theta}\right]^{\lambda_1} \iff \\
t_{kl} &= \bar{t}_{kl}^{\frac{1}{1+\theta\lambda_1}} \times P_k^{-\frac{\theta\lambda_1}{1+\theta\lambda_1}} \times \Pi_l^{-\frac{\theta\lambda_1}{1+\theta\lambda_1}}
\end{aligned}$$

Replacing,

$$\begin{aligned}
\Xi_{kl} &= \left(\bar{t}_{kl}^{\frac{1}{1+\theta\lambda_1}} \times P_k^{-\frac{\theta\lambda_1}{1+\theta\lambda_1}} \times \Pi_l^{-\frac{\theta\lambda_1}{1+\theta\lambda_1}} \right)^{-\theta} \times P_k^{-\theta} \times \Pi_l^{-\theta}, \\
&= \bar{t}_{kl}^{-\frac{\theta}{1+\theta\lambda_1}} \times P_k^{\frac{\theta\theta\lambda_1}{1+\theta\lambda_1}-\theta} \times \Pi_l^{\frac{\theta\lambda_1\theta}{1+\theta\lambda_1}-\theta} \\
&= \bar{t}_{kl}^{-\frac{\theta}{1+\theta\lambda_1}} \times P_k^{-\frac{\theta}{1+\theta\lambda_1}} \times \Pi_l^{-\frac{\theta}{1+\theta\lambda_1}}
\end{aligned}$$

therefore,

$$\Xi_{kl} = \bar{t}_{kl}^{-\frac{\theta}{1+\theta\lambda_1}} \times P_k^{-\frac{\theta}{1+\theta\lambda_1}} \times \Pi_l^{-\frac{\theta}{1+\theta\lambda_1}}$$

replacing the market access terms,

$$P_i = \frac{1}{\bar{W}} \bar{u}_i L_i^{\beta-1} Y_i$$

$$\Pi_i = \bar{A}_i L_i^{1+\alpha} Y_i^{-\frac{\theta+1}{\theta}}$$

we obtain,

$$\begin{aligned}
\Xi_{kl} &= \bar{t}_{kl}^{-\frac{\theta}{1+\theta\lambda_1}} \times \left(\frac{1}{\bar{W}} \bar{u}_k L_k^{\beta-1} Y_k \right)^{-\frac{\theta}{1+\theta\lambda_1}} \times \left(\bar{A}_l L_l^{1+\alpha} Y_l^{-\frac{\theta+1}{\theta}} \right)^{-\frac{\theta}{1+\theta\lambda_1}} \\
&= \bar{t}_{kl}^{-\frac{\theta}{1+\theta\lambda_1}} \left(\frac{\bar{L}^{-(\alpha+\beta)\theta}}{W^{-\theta}} \right)^{\frac{1}{1+\theta\lambda}} \bar{L}^{\frac{1}{1+\theta\lambda}} \bar{A}_l^{-\frac{\theta}{1+\theta\lambda}} \bar{u}_k^{-\frac{\theta}{1+\theta\lambda}} l_k^{-\frac{\theta(\beta-1)}{1+\theta\lambda}} l_l^{-\frac{\theta(1+\alpha)}{1+\theta\lambda}} y_k^{-\frac{\theta}{1+\theta\lambda}} y_l^{\frac{(1+\theta)}{1+\theta\lambda}} \\
&= \bar{t}_{kl}^{-\frac{\theta}{1+\theta\lambda_1}} \chi^{-\frac{1}{1+\theta\lambda}} \bar{L}^{\frac{1}{1+\theta\lambda}} \bar{A}_l^{-\frac{\theta}{1+\theta\lambda}} \bar{u}_k^{-\frac{\theta}{1+\theta\lambda}} l_k^{-\frac{\theta(\beta-1)}{1+\theta\lambda}} l_l^{-\frac{\theta(1+\alpha)}{1+\theta\lambda}} y_k^{-\frac{\theta}{1+\theta\lambda}} y_l^{\frac{(1+\theta)}{1+\theta\lambda}}
\end{aligned}$$

where in the last line we use the definition $\chi = \frac{\bar{L}^{(\alpha+\beta)\theta}}{W^{\theta}}$. We have,

$$\Xi_{kl} = \bar{t}_{kl}^{-\frac{\theta}{1+\theta\lambda_1}} \chi^{\frac{1}{1+\theta\lambda}} \bar{L}^{\frac{1}{1+\theta\lambda}} \bar{A}_l^{-\frac{\theta}{1+\theta\lambda}} \bar{u}_k^{-\frac{\theta}{1+\theta\lambda}} l_k^{-\frac{\theta(\beta-1)}{1+\theta\lambda}} l_l^{-\frac{\theta(1+\alpha)}{1+\theta\lambda}} y_k^{-\frac{\theta}{1+\theta\lambda}} y_l^{\frac{(1+\theta)}{1+\theta\lambda}}$$

A.3.2 Traffic on the secondary network

To begin, we characterize the expected number of times in which a route between (k', l') is used in trade between (i, j), π_{ij}^{kl}, which we refer to as, secondary mode intensity. Summing across all routes from i to j the product of the probability a particular route is used and the number of times that route passes through the secondary network (k', l'),

$$\pi_{ij}^{k'l'} \equiv \sum_{r\in\mathcal{R}_{ij}^{1,2}} \left(\frac{\pi_{ij,r}}{\sum_{r'\in\mathcal{R}_{ij}^{1}\cup\mathcal{R}_{ij}^{1,2}} \pi_{ij,r'}} \right) n_r^{k'l'}$$

Beginning with this equation,

$$\begin{aligned}
\pi_{ij}^{k'l'} &= \sum_{r\in\mathcal{R}_{ij}^{1,2}} \frac{\pi_{ij,r}}{\sum_{r'\in\mathcal{R}_{ij}^{1}\cup\mathcal{R}_{ij}^{1,2}} \pi_{ij,r'}} n_r^{k'l'} \iff \\
\pi_{ij}^{k'l'} &= \sum_{r\in\mathcal{R}_{ij}^{1,2}} \frac{\left(\prod_{l=1}^{K} t_{r_{l-1},r_l}^{-\theta}\right)}{\sum_{r'\in\mathcal{R}_{ij}^{1}\cup\mathcal{R}_{ij}^{1,2}} \left(\prod_{l=1}^{K} t_{r_{l-1},r_l}^{-\theta}\right)} n_r^{k'l'} \iff \\
\pi_{ij}^{k'l'} &= \tau_{ij}^{\theta} \sum_{r\in\mathcal{R}_{ij}^{1,2}} \left(\prod_{l=1}^{K} t_{r_{l-1},r_l}^{-\theta} \right) n_{r,}^{k'l'}
\end{aligned}$$

To calculate $\pi_{ij}^{k'l'}$, we proceed by summing across all possible traverses across the secondary network between k' and l' that occur on all routes from i to j. To do so, note first that this can be written in terms of two sets of routes. First, the routes that originate in i and reach k' by exclusively utilizing the primary network, and similarly the routes that for the final leg between l' and j exclusively use the primary network. Second, the routes that use a multimodal path for either the first leg (from i to k') or for the last leg (from l' to j). Finally, note that to do so for any route $r \in \mathcal{R}_{ij}^{1} \cup \mathcal{R}_{ij}^{1,2}$ of length K (which we denote as $\mathcal{R}_{ij,K}$), a traverse is possible at any point $B \in [1, 2, \ldots, K-1]$ in the route, which requires us to sum over all possible traverses at any point B, i.e.

$$\begin{aligned}
\pi_{ij}^{kl} = \frac{1}{b_{ij}} \sum_{K=0}^{\infty} \sum_{B=0}^{K-1} & \left(\sum_{r\in\mathcal{R}_{ik,B}^{1}\cup\mathcal{R}_{ik,B}^{1,2}} \prod_{n=1}^{B} a_{r_{n-1},r_n} \right) \times a_{kk'} \\
& \times \left(\sum_{L=0}^{\infty} \sum_{r\in\mathcal{R}_{k'l'}^{1,2}\cup\mathcal{R}_{k'l'}^{2}} \prod_{n=1}^{L} a_{r_{n-1},r_n} \right) \times a_{l'l} \\
& \times \left(\sum_{r\in\mathcal{R}_{lj,K-B-1}^{1,2}\cup\mathcal{R}_{lj,K-B-1}^{1}} \prod_{n=1}^{K-B-1} a_{r_{n-1},r_n} \right)
\end{aligned}$$

where in the middle bracket we have enumerated all the multi- and unimodal paths along the secondary network that traverse between k' and l' and has any possible route length between 0 and ∞.

Explicitly enumerating all possible paths,

$$\pi_{ij}^{k'l'} = \frac{1}{b_{ij}} \sum_{K=0}^{\infty} \sum_{B=0}^{K-1} \left(\sum_{n_1=1}^{(N_1+N_2)} \cdots \sum_{n_{B-1}=1}^{(N_1+N_2)} a_{i,n_1} \times \ldots \times a_{n_{B-1},k} \right),$$
$$\times a_{kk'} \times \left(\sum_{L=0}^{\infty} \left(\sum_{k_1=1}^{(N_1+N_2)} \sum_{k_2=1}^{(N_1+N_2)} \cdots \sum_{k_{L-1}=1}^{(N_1+N_2)} a_{k',k_1} \times a_{k_1,k_2} \times \ldots \times a_{k_{K-2},k_{K-1}} \times a_{k_{K-1},l'} \right) \right) \times a_{l'l}$$
$$\times \left(\sum_{n_1=1}^{(N_1+N_2)} \cdots \sum_{n_{K-B-1}=1}^{(N_1+N_2)} a_{l,n_1} \times \ldots \times a_{n_{K-B-1},j} \right)$$

which can be exporessed more succinctly as elements of matrix powers of A:

$$\pi_{ij}^{k'l'} = \frac{1}{b_{ij}} \sum_{K=0}^{\infty} \sum_{B=0}^{K-1} \mathbf{A}_{ik}^{B} \times a_{kk'} \times \left(\sum_{K=0}^{\infty} \mathbf{A}_{k'l'}^{K} \right) \times a_{ll'} \times \mathbf{A}_{lj}^{K-B-1}$$

A result from matrix calculus is for any $N \times N$ matrix we have:

$$\sum_{K=0}^{\infty} \sum_{B=0}^{K-1} \mathbf{A}^{B} \mathbf{C} \mathbf{A}^{K-B-1} = (\mathbf{I} - \mathbf{A})^{-1} \mathbf{C} (\mathbf{I} - \mathbf{A})^{-1}$$

Recall that

$$\sum_{K=0}^{\infty} \mathbf{A}_{k'l'}^{K} = (\mathbf{I} - \mathbf{A})^{-1} \equiv \mathbf{F}$$

and define the transportation cost along the secondary transportation network as,

$$\tau_{k'l'} = f_{k'l'}^{-\frac{1}{\theta}}$$

Define C to be an $N \times N$ matrix, $\mathbf{C} \equiv [c_{kl}] = \left[s_{kk'}^{-\theta} \tau_{k'l'}^{-\theta} s_{l'l}^{-\theta} \right]$ row k and column l and zeros everywhere else. We obtain:

$$\pi_{ij}^{k'l'} = \frac{\tau_{ik}^{-\theta} s_{kk'}^{-\theta} \tau_{k'l'}^{-\theta} s_{l'l}^{-\theta} \tau_{lj}^{-\theta}}{\tau_{ij}^{-\theta}}$$

We now derive the gravity equations for traffic over a link. For trade, we sum over all trade between all origins and desitnations, and all routes taken by that trade, to get:

$$\Xi_{k'l'} \equiv \Xi_{kl}^{2} = \sum_{i \in \mathcal{N}} \sum_{j \in \mathcal{N}} \sum_{r \in \mathcal{R}_{ij}^{1;2}} \pi_{ij,r} n_r^{kl} E_j \iff$$
$$\Xi_{kl}^{2} = \sum_{i \in \mathcal{N}} \sum_{j \in \mathcal{N}} \pi_{ij}^{k'l'} X_{ij} \iff$$
$$\Xi_{kl}^{2} = \sum_{i \in \mathcal{N}} \sum_{j \in \mathcal{N}} \frac{\tau_{ik}^{-\theta} s_{kk'}^{-\theta} \tau_{k'l'}^{-\theta} s_{l'l}^{-\theta} \tau_{lj}^{-\theta}}{\tau_{ij}^{-\theta}} \times \tau_{ij}^{-\theta} \frac{Y_i}{\Pi_i^{-\theta}} \frac{E_j}{P_j^{-\theta}} \iff$$
$$\Xi_{kl}^{2} = s_{kk'}^{-\theta} \tau_{k'l'}^{-\theta} s_{l'l}^{-\theta} \sum_{i \in \mathcal{N}} \tau_{ik}^{-\theta} \frac{Y_i}{\Pi_i^{-\theta}} \sum_{j \in \mathcal{N}} \tau_{lj}^{-\theta} \frac{E_j}{P_j^{-\theta}},$$

Recalling the definition of the consumer and producer market access terms,

$$\Pi_i \equiv \left(\sum_{j=1}^{N} \tau_{ij}^{-\theta} E_j P_j^{\theta}\right)^{-\frac{1}{\theta}} = A_i L_i Y_i^{-\frac{\theta+1}{\sigma}}$$

$$P_j = \left(\sum_{i=1}^{N} \tau_{ij}^{-\theta} Y_i \Pi_i^{\theta}\right)^{-\frac{1}{\theta}}$$

we get,

$$\Xi_{kl}^2 = s_{kk'}^{-\theta} \tau_{k'l'}^{-\theta} s_{l'l}^{-\theta} \times P_k^{-\theta} \times \Pi_l^{-\theta},$$

replacing the market access terms,

$$P_i = \frac{1}{\bar{W}} \bar{u}_i L_i^{\beta-1} Y_i$$

$$\Pi_i = \bar{A}_i L_i^{1+\alpha} Y_i^{-\frac{\theta+1}{\theta}}$$

we obtain,

$$\begin{aligned}
\Xi_{kl}^2 &= s_{kk'}^{-\theta} \tau_{k'l'}^{-\theta} s_{l'l}^{-\theta} \times \left(\frac{1}{\bar{W}} \bar{u}_k L_k^{\beta-1} Y_k\right)^{-\theta} \times \left(\bar{A}_l L_l^{1+\alpha} Y_l^{-\frac{\theta+1}{\theta}}\right)^{-\theta} \\
&= s_{kk'}^{-\theta} \tau_{k'l'}^{-\theta} s_{l'l}^{-\theta} \left(\frac{\bar{L}^{-(\alpha+\beta)\theta}}{W^{-\theta}}\right) \bar{L} \bar{A}_l^{-\theta} \bar{u}_k^{-\theta} l_k^{-\theta(\beta-1)} l_l^{-\theta(1+\alpha)} y_k^{-\theta} y_l^{(1+\theta)} \\
&= s_{kk'}^{-\theta} \tau_{k'l'}^{-\theta} s_{l'l}^{-\theta} \chi \bar{L} \bar{A}_l^{-\theta} \bar{u}_k^{-\theta} l_k^{-\theta(\beta-1)} l_l^{-\theta(1+\alpha)} y_k^{-\theta} y_l^{(1+\theta)}
\end{aligned}$$

where in the last line we use the definition $\chi = \frac{\bar{L}^{(\alpha+\beta)\theta}}{W^{\theta}}$. We have,

$$\Xi_{kl}^2 = s_{kk'}^{-\theta} \tau_{k'l'}^{-\theta} s_{l'l}^{-\theta} \chi^{-1} \bar{L} \bar{A}_l^{-\theta} \bar{u}_k^{-\theta} l_k^{-\theta} l_l^{-\theta} y_k^{-\theta} y_l^{(1+\theta)}$$

A.3.3 Traffic at terminals

To begin, we characterize the expected number of times in which a route between (k', l') is used in trade between (i, j), π_{ij}^{kl}, which we refer to as, secondary mode intensity. Summing across all routes from i to j the product of the probability a particular route is used and the number of times that route passes through the secondary network (k', l'),

$$\pi_{ij}^{kk'} \equiv \sum_{r \in \mathcal{R}_{ij}^{1,2}} \left(\frac{\pi_{ij,r}}{\sum_{r' \in \mathcal{R}_{ij}^{1} \cup \mathcal{R}_{ij}^{1,2}} \pi_{ij,r'}}\right) n_r^{kk'}$$

Beginning with this equation,

$$\pi_{ij}^{kk'} = \sum_{r\in\mathcal{R}_{ij}^{1,2}} \frac{\pi_{ij,r}}{\sum_{r'\in\mathcal{R}_{ij}^{1}\cup\mathcal{R}_{ij}^{1,2}} \pi_{ij,r'}} n_r^{kk'} \iff$$

$$\pi_{ij}^{kk'} = \sum_{r\in\mathcal{R}_{ij}^{1,2}} \frac{\left(\prod_{l=1}^{K} t_{r_{l-1},r_l}^{-\theta}\right)}{\sum_{r'\in\mathcal{R}_{ij}^{1}\cup\mathcal{R}_{ij}^{1,2}} \left(\prod_{l=1}^{K} t_{r_{l-1},r_l}^{-\theta}\right)} n_r^{kk'} \iff$$

$$\pi_{ij}^{kk'} = \tau_{ij}^{\theta} \sum_{r\in\mathcal{R}_{ij}^{1,2}} \left(\prod_{l=1}^{K} t_{r_{l-1},r_l}^{-\theta}\right) n_{r,}^{kk'}$$

To calculate $\pi_{ij}^{kk'}$, we proceed by summing across all possible traverses across the secondary network that through the terminal station $\{k,k'\}$. This in turn requires characterizing all possible routes that use the secondary transportation network along the path from i to j and traverse through terminal node $\{k,k'\}$ with any possible end point along the secondary transportation network. To do so, note first that this can be written in terms of two sets of routes. First, the routes that originate in i and reach k' by exclusively utilizing the primary network, and similarly the routes that for the final leg between **any** l' and j exclusively use the primary network. Second, the routes that use a multimodal path for either the first leg (from i to k') or for the last leg (from l' to j). Finally, note that to do so for any route a traverse is possible with any multimodal route that has an arbitrary end point on the secondary network (l') where we have to explicitly sum over all possible end points. As before, for any route $r \in \mathcal{R}_{ij}^{1} \cup \mathcal{R}_{ij}^{1,2}$ of length K (which we denote as $\mathcal{R}_{ij,K}$), a traverse is possible at any point $B \in [1,2,\ldots,K-1]$ in the route, which requires us to sum over all possible traverses at any point B, i.e.

$$\pi_{ij}^{kk'} = \frac{1}{b_{ij}} \sum_{K=0}^{\infty}\sum_{B=0}^{K-1} \left(\sum_{r\in\mathcal{R}_{ik,B}^{1}\cup\mathcal{R}_{ik,B}^{1,2}} \prod_{n=1}^{B} a_{r_{n-1},r_n} \right) \times a_{kk'}$$
$$\times \left\{ \sum_{l'=0}^{N_2} \left(\sum_{L=0}^{\infty} \sum_{r\in\mathcal{R}_{k'l'}^{1,2}\cup\mathcal{R}_{k'l'}^{2}} \prod_{n=1}^{L} a_{r_{n-1},r_n} \right) \times a_{l'l} \times \left(\sum_{r\in\mathcal{R}_{lj,K-B-1}^{1,2}\cup\mathcal{R}_{lj,K-B-1}^{1}} \prod_{n=1}^{K-B-1} a_{r_{n-1},r_n} \right) \right\}$$

where in the second line, in the curly brackets, we sum explicitly over all possible end points l' that the multimodal path might along the secondary network might take. Explicitly enumerating all possible paths,

$$\pi_{ij}^{kk'} = \frac{1}{b_{ij}} \sum_{K=0}^{\infty}\sum_{B=0}^{K-1} \left(\sum_{n_1=1}^{(N_1+N_2)} \cdots \sum_{n_{B-1}=1}^{(N_1+N_2)} a_{i,n_1} \times \ldots \times a_{n_{B-1},k} \right) \times a_{kk'}$$
$$\times (\sum_{l'=0}^{N_2} \left(\sum_{L=0}^{\infty} \left(\sum_{k_1=1}^{(N_1+N_2)} \sum_{k_2=1}^{(N_1+N_2)} \cdots \sum_{k_{L-1}=1}^{(N_1+N_2)} a_{k',k_1} \times a_{k_1,k_2} \times \ldots \times a_{k_{K-2},k_{K-1}} \times a_{k_{K-1},l'} \right) \right)$$
$$\times a_{l'l} \times \left(\sum_{n_1=1}^{(N_1+N_2)} \cdots \sum_{n_{K-B-1}=1}^{(N_1+N_2)} a_{l,n_1} \times \ldots \times a_{n_{K-B-1},j} \right))$$

which can be exporessed more succinctly as elements of matrix powers of A:

$$\begin{aligned}
\pi_{ij}^{k'l'} &= \frac{1}{b_{ij}} \sum_{K=0}^{\infty} \sum_{B=0}^{K-1} \mathbf{A}_{ik}^{B} \times a_{kk'} \times \left(\left(\sum_{K=0}^{\infty} \mathbf{A}^{K} \right) \times \mathbf{S}' \times \mathbf{A}^{K-B-1} \right)_{k'j} \\
&= \frac{1}{b_{ij}} \sum_{K=0}^{\infty} \sum_{B=0}^{K-1} \mathbf{A}_{ik}^{B} \times a_{kk'} \times \left((\mathbf{I}-\mathbf{A})^{-1} \times \mathbf{S}' \times \mathbf{A}^{K-B-1} \right)_{k'j}
\end{aligned}$$

defining $\sum_{K=0}^{\infty} \mathbf{A}_{k'l'}^{K} = (\mathbf{I}-\mathbf{A})^{-1} \equiv \mathbf{F}$,

$$\pi_{ij}^{k'l'} = \frac{1}{b_{ij}} \sum_{K=0}^{\infty} \sum_{B=0}^{K-1} \mathbf{A}_{ik}^{B} \times a_{kk'} \times \left(\mathbf{F} \times \mathbf{S}' \times \mathbf{A}^{K-B-1} \right)_{k'j}$$

again in summation formulation,

$$\begin{aligned}
\pi_{ij}^{kk'} &= \frac{1}{b_{ij}} \sum_{K=0}^{\infty} \sum_{B=0}^{K-1} \left(\sum_{n_1=1}^{(N_1+N_2)} \cdots \sum_{n_{B-1}=1}^{(N_1+N_2)} a_{i,n_1} \times \ldots \times a_{n_{B-1},k} \right) \times a_{kk'} \\
&\quad \times \left\{ \sum_{l'=0}^{N_2} f_{k'l'} \times a_{l'l} \times \left(\sum_{n_1=1}^{(N_1+N_2)} \cdots \sum_{n_{K-B-1}=1}^{(N_1+N_2)} a_{l,n_1} \times \ldots \times a_{n_{K-B-1},j} \right) \right\}
\end{aligned}$$

A result from matrix calculus is for any $N \times N$ matrix we have:

$$\sum_{K=0}^{\infty} \sum_{B=0}^{K-1} \mathbf{A}^{B} \mathbf{C} \mathbf{A}^{K-B-1} = (\mathbf{I}-\mathbf{A})^{-1} \mathbf{C} (\mathbf{I}-\mathbf{A})^{-1}$$

Define C to be an $N \times N$ matrix that results from a matrix multiplication, $\mathbf{C} = \mathbf{GH}$, where $\mathbf{G}$ is a diagonal matrix where the off-diagonal elements are zero and the diagonal elements correspond to the switching cost, i.e. $\mathbf{G} = [g_{kk'}] = \left[s_{kk'}^{-\theta} \right]$, and where $\mathbf{H} = [h_{kl}] = \left[\sum_l \tau_{k'l'}^{-\theta} s_{l'l}^{-\theta} \tau_{lj}^{-\theta} \right]$ finally, the matrix $\mathbf{C}$ is then given by, $\mathbf{C} \equiv [c_{kj}] = \left[s_{kk'}^{-\theta} \left(\sum_l \tau_{k'l'}^{-\theta} s_{l'l}^{-\theta} \tau_{lj}^{-\theta} \right) \right]$. We obtain:

$$\pi_{ij}^{kk'} = \frac{\tau_{ik}^{-\theta} s_{kk'}^{-\theta} \left(\sum_l \tau_{k'l'}^{-\theta} s_{l'l}^{-\theta} \tau_{lj}^{-\theta} \right)}{\tau_{ij}^{-\theta}}$$

We now derive the gravity equations for traffic over a terminal station. For trade, we sum over all trade between all origins and desitnations, and all routes taken by that trade, to get:

$$
\begin{aligned}
\Xi^2_{kk'} &= \sum_{i\in\mathcal{N}}\sum_{j\in\mathcal{N}}\sum_{r\in\mathcal{R}^{1,2}_{ij}} \pi_{ij,r} n^{kk'}_r E_j \iff \\
\Xi^2_{kk'} &= \sum_{i\in\mathcal{N}}\sum_{j\in\mathcal{N}} \pi^{kk'}_{ij} X_{ij} \iff \\
\Xi^2_{kk'} &= \sum_{i\in\mathcal{N}}\sum_{j\in\mathcal{N}} \frac{\tau^{-\theta}_{ik} s^{-\theta}_{kk'} \left(\sum_l \tau^{-\theta}_{k'l'} s^{-\theta}_{l'l} \tau^{-\theta}_{lj}\right)}{\tau^{-\theta}_{ij}} \times \tau^{-\theta}_{ij} \frac{Y_i}{\Pi^{-\theta}_i} \frac{E_j}{P^{-\theta}_j} \iff \\
\Xi^2_{kk'} &= s^{-\theta}_{kk'} \sum_{i\in\mathcal{N}} \tau^{-\theta}_{ik} \frac{Y_i}{\Pi^{-\theta}_i} \sum_{j\in\mathcal{N}} \left(\sum_l \tau^{-\theta}_{k'l'} s^{-\theta}_{l'l} \tau^{-\theta}_{lj}\right) \frac{E_j}{P^{-\theta}_j}, \\
\Xi^2_{kk'} &= s^{-\theta}_{kk'} \sum_{i\in\mathcal{N}} \tau^{-\theta}_{ik} \frac{Y_i}{\Pi^{-\theta}_i} \sum_l \tau^{-\theta}_{k'l'} s^{-\theta}_{l'l} \sum_{j\in\mathcal{N}} \left(\tau^{-\theta}_{lj}\right) \frac{E_j}{P^{-\theta}_j},
\end{aligned}
$$

Recalling the definition of the consumer and producer market access terms,

$$
\Pi_i \equiv \left(\sum_{j=1}^N \tau^{-\theta}_{ij} E_j P^{\theta}_j\right)^{-\frac{1}{\theta}} = A_i L_i Y_i^{-\frac{\theta+1}{\sigma}}
$$

$$
P_j = \left(\sum_{i=1}^N \tau^{-\theta}_{ij} Y_i \Pi^{\theta}_i\right)^{-\frac{1}{\theta}}
$$

we get,

$$
\begin{aligned}
\Xi^2_{kk'} &= s^{-\theta}_{kk'} \sum_{i\in\mathcal{N}} \tau^{-\theta}_{ik} \frac{Y_i}{\Pi^{-\theta}_i} \sum_l \tau^{-\theta}_{k'l'} s^{-\theta}_{l'l} \sum_{j\in\mathcal{N}} \left(\tau^{-\theta}_{lj}\right) \frac{E_j}{P^{-\theta}_j}, \\
&= s^{-\theta}_{kk'} P^{-\theta}_k \sum_l \tau^{-\theta}_{k'l'} s^{-\theta}_{l'l} \Pi^{-\theta}_l
\end{aligned}
$$

$$
\Xi^2_{kk'} = (s_{kk'})^{-\theta} \times P^{-\theta}_k \times \sum_l \tau^{-\theta}_{k'l'} s^{-\theta}_{l'l} \Pi^{-\theta}_l,
$$

Substituting with the functional form for congestion,

$$
\begin{aligned}
s_{kk'} &= \bar{s}_{kk'} \left[\Xi_{kk'}\right]^{\lambda_2} \iff \\
s_{kk'} &= \bar{s}_{kk'} \left[(s_{kk'})^{-\theta} \times P^{-\theta}_k \times \sum_l \tau^{-\theta}_{k'l'} s^{-\theta}_{l'l} \Pi^{-\theta}_l\right]^{\lambda_2} \iff \\
s_{kk'} &= \bar{s}_{kk'}^{\frac{1}{1+\theta\lambda_2}} \times P_k^{-\frac{\theta\lambda_2}{1+\theta\lambda_2}} \times \left(\sum_l \tau^{-\theta}_{k'l'} s^{-\theta}_{l'l} \Pi^{-\theta}_l\right)^{\frac{\lambda_2}{1+\theta\lambda_2}}
\end{aligned}
$$

Replacing,

$$\Xi_{kk'}^2 = \left(\bar{s}_{kk'}^{\frac{1}{1+\theta\lambda_2}} \times P_k^{-\frac{\theta\lambda_2}{1+\theta\lambda_2}} \times \left(\sum_l \tau_{k'l'}^{-\theta} s_{l'l}^{-\theta} \Pi_l^{-\theta} \right)^{\frac{\lambda_2}{1+\theta\lambda_2}} \right)^{-\theta} \times P_k^{-\theta} \times \sum_l \tau_{k'l'}^{-\theta} s_{l'l}^{-\theta} \Pi_l^{-\theta},$$

$$= \bar{s}_{kk'}^{-\frac{\theta}{1+\theta\lambda_2}} \times P_k^{\frac{\theta\theta\lambda_2}{1+\theta\lambda_2}-\theta} \times \left(\sum_l \tau_{k'l'}^{-\theta} s_{l'l}^{-\theta} \Pi_l^{-\theta} \right)^{-\frac{\theta\lambda_2}{1+\theta\lambda_2}+1}$$

$$= \bar{s}_{kk'}^{-\frac{\theta}{1+\theta\lambda_2}} \times P_k^{-\frac{\theta}{1+\theta\lambda_2}} \times \left(\sum_l \tau_{k'l'}^{-\theta} s_{l'l}^{-\theta} \Pi_l^{-\theta} \right)^{\frac{1}{1+\theta\lambda_2}}$$

therefore,

$$\Xi_{kk'}^2 = \bar{s}_{kk'}^{-\frac{\theta}{1+\theta\lambda_2}} \times P_k^{-\frac{\theta}{1+\theta\lambda_2}} \times \left(\sum_l \tau_{k'l'}^{-\theta} s_{l'l}^{-\theta} \Pi_l^{-\theta} \right)^{\frac{1}{1+\theta\lambda_2}}$$

Deriving symmetrically for traffic flows into the opposite direction,

$$\pi_{ij}^{k'k} \equiv \sum_{r \in \mathcal{R}_{ij}^{1,2}} \left(\frac{\pi_{ij,r}}{\sum_{r' \in \mathcal{R}_{ij}^1 \cup \mathcal{R}_{ij}^{1,2}} \pi_{ij,r'}} \right) n_r^{k'k}$$

Following the same steps as above we obtain,

$$\pi_{ij}^{k'k} = \frac{\left(\sum_l \tau_{il}^{-\theta} s_{ll'}^{-\theta} \tau_{l'k'}^{-\theta} \right) s_{k'k}^{-\theta} \tau_{kj}^{-\theta}}{\tau_{ij}^{-\theta}}$$

Summing over all trade and substituting,

$$\Xi_{k'k}^2 = \sum_{i \in \mathcal{N}} \sum_{j \in \mathcal{N}} \sum_{r \in \mathcal{R}_{ij}^{1,2}} \pi_{ij,r} n_r^{k'k} E_j \iff$$

$$\Xi_{k'k}^2 = \sum_{i \in \mathcal{N}} \sum_{j \in \mathcal{N}} \pi_{ij}^{k'k} X_{ij} \iff$$

$$\Xi_{k'k}^2 = \sum_{i \in \mathcal{N}} \sum_{j \in \mathcal{N}} \frac{\left(\sum_l \tau_{il}^{-\theta} s_{ll'}^{-\theta} \tau_{l'k'}^{-\theta} \right) s_{k'k}^{-\theta} \tau_{kj}^{-\theta}}{\tau_{ij}^{-\theta}} \times \tau_{ij}^{-\theta} \frac{Y_i}{\Pi_i^{-\theta}} \frac{E_j}{P_j^{-\theta}} \iff$$

$$\Xi_{k'k}^2 = s_{k'k}^{-\theta} \sum_{j \in \mathcal{N}} \tau_{kj}^{-\theta} \frac{E_j}{P_j^{-\theta}} \sum_{i \in \mathcal{N}} \left(\sum_l \tau_{il}^{-\theta} s_{ll'}^{-\theta} \tau_{l'k'}^{-\theta} \right) \frac{Y_i}{\Pi_i^{-\theta}},$$

$$\Xi_{k'k}^2 = s_{k'k}^{-\theta} \sum_{j \in \mathcal{N}} \tau_{kj}^{-\theta} \frac{E_j}{P_j^{-\theta}} \sum_l s_{ll'}^{-\theta} \tau_{l'k'}^{-\theta} \sum_{i \in \mathcal{N}} \left(\tau_{il}^{-\theta} \right) \frac{Y_i}{\Pi_i^{-\theta}},$$

Recalling the definition of the consumer and producer market access terms,

$$\Pi_i \equiv \left(\sum_{j=1}^N \tau_{ij}^{-\theta} E_j P_j^{\theta} \right)^{-\frac{1}{\theta}} = A_i L_i Y_i^{-\frac{\theta+1}{\sigma}}$$

$$P_j = \left(\sum_{i=1}^{N} \tau_{ij}^{-\theta} Y_i \Pi_i^{\theta}\right)^{-\frac{1}{\theta}}$$

we get,

$$\begin{aligned}
\Xi_{k'k}^2 &= s_{k'k}^{-\theta} \sum_{j\in\mathcal{N}} \tau_{kj}^{-\theta} \frac{E_j}{P_j^{-\theta}} \sum_l s_{ll'}^{-\theta} \tau_{l'k'}^{-\theta} \sum_{i\in\mathcal{N}} \left(\tau_{il}^{-\theta}\right) \frac{Y_i}{\Pi_i^{-\theta}}, \\
&= s_{k'k}^{-\theta} \Pi_k^{-\theta} \sum_l s_{ll'}^{-\theta} \tau_{l'k'}^{-\theta} P_l^{-\theta}
\end{aligned}$$

$$\Xi_{k'k}^2 = (s_{k'k})^{-\theta} \times \Pi_k^{-\theta} \times \sum_l s_{ll'}^{-\theta} \tau_{l'k'}^{-\theta} P_l^{-\theta},$$

Substituting with the functional form for congestion,

$$\begin{aligned}
s_{k'k} &= \bar{s}_{k'k} \left[\Xi_{k'k}\right]^{\lambda_2} \iff \\
s_{k'k} &= \bar{s}_{k'k} \left[(s_{k'k})^{-\theta} \times \Pi_k^{-\theta} \times \sum_l s_{ll'}^{-\theta} \tau_{l'k'}^{-\theta} P_l^{-\theta}\right]^{\lambda_2} \iff \\
s_{k'k} &= \bar{s}_{k'k}^{\frac{1}{1+\theta\lambda_2}} \times \Pi_k^{-\frac{\theta\lambda_2}{1+\theta\lambda_2}} \times \left(\sum_l s_{ll'}^{-\theta} \tau_{l'k'}^{-\theta} P_l^{-\theta}\right)^{\frac{\lambda_2}{1+\theta\lambda_2}}
\end{aligned}$$

$$s_{l'l} = \bar{s}_{l'l}^{\frac{1}{1+\theta\lambda_2}} \times \Pi_l^{-\frac{\theta\lambda_2}{1+\theta\lambda_2}} \times \left(\sum_k s_{kk'}^{-\theta} \tau_{k'l'}^{-\theta} P_k^{-\theta}\right)^{\frac{\lambda_2}{1+\theta\lambda_2}}$$

Replacing,

$$\begin{aligned}
\Xi_{k'k}^2 &= \left(\bar{s}_{k'k}^{\frac{1}{1+\theta\lambda_2}} \times \Pi_k^{-\frac{\theta\lambda_2}{1+\theta\lambda_2}} \times \left(\sum_l s_{ll'}^{-\theta} \tau_{l'k'}^{-\theta} P_l^{-\theta}\right)^{\frac{\lambda_2}{1+\theta\lambda_2}}\right)^{-\theta} \times \Pi_k^{-\theta} \times \left(\sum_l s_{ll'}^{-\theta} \tau_{l'k'}^{-\theta} P_l^{-\theta}\right) \\
&= \bar{s}_{k'k}^{-\frac{\theta}{1+\theta\lambda_2}} \times \Pi_k^{\frac{\theta\theta\lambda_2}{1+\theta\lambda_2}-\theta} \times \left(\sum_l s_{ll'}^{-\theta} \tau_{l'k'}^{-\theta} P_l^{-\theta}\right)^{-\frac{\theta\lambda_2}{1+\theta\lambda_2}+1} \\
&= \bar{s}_{k'k}^{-\frac{\theta}{1+\theta\lambda}} \times \Pi_k^{-\frac{\theta}{1+\theta\lambda_2}} \times \left(\sum_l s_{ll'}^{-\theta} \tau_{l'k'}^{-\theta} P_l^{-\theta}\right)^{\frac{1}{1+\theta\lambda_2}}
\end{aligned}$$

therefore,

$$\Xi_{k'k}^2 = \bar{s}_{k'k}^{-\frac{\theta}{1+\theta\lambda_2}} \times \Pi_k^{-\frac{\theta}{1+\theta\lambda_2}} \times \left(\sum_l s_{ll'}^{-\theta} \tau_{l'k'}^{-\theta} P_l^{-\theta}\right)^{\frac{1}{1+\theta\lambda_2}}$$

$$\Xi_{l'l}^2 = \bar{s}_{l'l}^{-\frac{\theta}{1+\theta\lambda_2}} \times \Pi_l^{-\frac{\theta}{1+\theta\lambda_2}} \times \left(\sum_k s_{kk'}^{-\theta} \tau_{k'l'}^{-\theta} P_k^{-\theta}\right)^{\frac{1}{1+\theta\lambda_2}}$$

$$\Xi_{kk'}^2 = \bar{s}_{kk'}^{-\frac{\theta}{1+\theta\lambda_2}} \times P_k^{-\frac{\theta}{1+\theta\lambda_2}} \times \left(\sum_l \tau_{k'l'}^{-\theta} s_{l'l}^{-\theta} \Pi_l^{-\theta}\right)^{\frac{1}{1+\theta\lambda_2}}$$

and replacing the equilibrium flows on the secondary transportation infrastructure,

$$\begin{aligned}
\Xi^2_{kl} &= \left(s_{kk'}\right)^{-\theta}\tau^{-\theta}_{k'l'}\left(s_{l'l}\right)^{-\theta}\times P_k^{-\theta}\times\Pi_l^{-\theta},\\
&= \left(\bar{s}_{kk'}^{\frac{1}{1+\theta\lambda_2}}\times P_k^{-\frac{\theta\lambda_2}{1+\theta\lambda_2}}\times\left(\sum_l \tau^{-\theta}_{k'l'}s^{-\theta}_{l'l}\Pi_l^{-\theta}\right)^{\frac{\lambda_2}{1+\theta\lambda_2}}\right)^{-\theta}\tau^{-\theta}_{k'l'}\left(\bar{s}_{l'l}^{\frac{1}{1+\theta\lambda_2}}\times\Pi_l^{-\frac{\theta\lambda_2}{1+\theta\lambda_2}}\times\left(\sum_k s^{-\theta}_{kk'}\tau^{-\theta}_{k'l'}P_k^{-\theta}\right)^{\frac{\lambda_2}{1+\theta\lambda_2}}\right)^{-\theta}\times P\\
&= \bar{s}_{kk'}^{-\frac{\theta}{1+\theta\lambda_2}}\times P_k^{\frac{\theta\theta\lambda_2}{1+\theta\lambda_2}-\theta}\times\left(\sum_l \tau^{-\theta}_{k'l'}s^{-\theta}_{l'l}\Pi_l^{-\theta}\right)^{-\frac{\theta\lambda_2}{1+\theta\lambda_2}}\times\bar{s}_{ll'}^{-\frac{\theta}{1+\theta\lambda_2}}\times\Pi_l^{\frac{\theta\theta\lambda_2}{1+\theta\lambda_2}-\theta}\times\left(\sum_k s^{-\theta}_{kk'}\tau^{-\theta}_{k'l'}P_k^{-\theta}\right)^{-\frac{\theta\lambda_2}{1+\theta\lambda_2}}\times\tau^{-\theta}_{k'l'}\\
&= \bar{s}_{kk'}^{-\frac{\theta}{1+\theta\lambda_2}}\times P_k^{-\frac{\theta}{1+\theta\lambda_2}}\times\left(\sum_l \tau^{-\theta}_{k'l'}s^{-\theta}_{l'l}\Pi_l^{-\theta}\right)^{-\frac{\theta\lambda_2}{1+\theta\lambda_2}}\times\bar{s}_{ll'}^{-\frac{\theta}{1+\theta\lambda_2}}\times\Pi_l^{-\frac{\theta}{1+\theta\lambda_2}}\times\left(\sum_k s^{-\theta}_{kk'}\tau^{-\theta}_{k'l'}P_k^{-\theta}\right)^{-\frac{\theta\lambda_2}{1+\theta\lambda_2}}\times\tau^{-\theta}_{k'l'}
\end{aligned}$$

in summary, equilibrium traffic flows are given by,

$$\Xi^2_{kk'} = \bar{s}_{kk'}^{-\frac{\theta}{1+\theta\lambda_2}}\times P_k^{-\frac{\theta}{1+\theta\lambda_2}}\times\left(\sum_l \tau^{-\theta}_{k'l'}s^{-\theta}_{l'l}\Pi_l^{-\theta}\right)^{\frac{1}{1+\theta\lambda_2}}$$

$$\Xi^2_{k'k} = \bar{s}_{k'k}^{-\frac{\theta}{1+\theta\lambda_2}}\times \Pi_k^{-\frac{\theta}{1+\theta\lambda_2}}\times\left(\sum_l s^{-\theta}_{ll'}\tau^{-\theta}_{l'k'}P_l^{-\theta}\right)^{\frac{1}{1+\theta\lambda_2}}$$

$$\Xi^2_{kl} = \bar{s}_{kk'}^{-\frac{\theta}{1+\theta\lambda_2}}\times P_k^{-\frac{\theta}{1+\theta\lambda_2}}\times\left(\sum_l \tau^{-\theta}_{k'l'}s^{-\theta}_{l'l}\Pi_l^{-\theta}\right)^{-\frac{\theta\lambda_2}{1+\theta\lambda_2}}\times\bar{s}_{l'l}^{-\frac{\theta}{1+\theta\lambda_2}}\times\Pi_l^{-\frac{\theta}{1+\theta\lambda_2}}\times\left(\sum_k s^{-\theta}_{kk'}\tau^{-\theta}_{k'l'}P_k^{-\theta}\right)^{-\frac{\theta\lambda_2}{1+\theta\lambda_2}}\times\tau^{-\theta}_{k'l'}$$

And equilibrium switching costs are given by,

$$s_{k'k} = \bar{s}_{k'k}^{\frac{1}{1+\theta\lambda_2}}\times \Pi_k^{-\frac{\theta\lambda_2}{1+\theta\lambda_2}}\times\left(\sum_l s^{-\theta}_{ll'}\tau^{-\theta}_{l'k'}P_l^{-\theta}\right)^{\frac{\lambda_2}{1+\theta\lambda_2}}$$

$$s_{kk'} = \bar{s}_{kk'}^{\frac{1}{1+\theta\lambda_2}}\times P_k^{-\frac{\theta\lambda_2}{1+\theta\lambda_2}}\times\left(\sum_l \tau^{-\theta}_{k'l'}s^{-\theta}_{l'l}\Pi_l^{-\theta}\right)^{\frac{\lambda_2}{1+\theta\lambda_2}}$$

A.4 Section 3.3.4: General Equilibrium with Traffic

Start with equation 8,

$$\bar{A}_i^{-\theta}y_i^{1+\theta}l_i^{-\theta(1+\alpha)} = \chi\sum_{j=1}^{N}\tau_{ij}^{-\theta}\bar{u}_j^{\theta}y_j{}^{1+\theta}l_j^{\theta(\beta-1)}$$

Note that multimodal routing implies that the transportation cost is given by, $\tau_{ij}^{-\theta} = \left[\left(\mathbf{I}-\mathbf{A_1}\right)-\mathbf{S}\left(\mathbf{I}-\mathbf{A_2}\right)^{-1}\mathbf{S'}\right]_{i}$ where $\mathbf{A_1} = [a_{ij}] = \left[t_{ij}^{-\theta}\right]$ is the adjacency matrix for the primary transportation network, $\mathbf{A_2} = \left[a_{i'j'}\right] = \left[t_{i'j'}^{-\theta}\right]$ is the adjacency matrix for the secondary transportation network, and $\mathbf{S} = [s_{ii'}]$ is the diagonal matrix that represents linkages between the primary and secondary transportation network. We can write,

$$\left[\bar{A}_i^{-\theta} y_i^{1+\theta} l_i^{-\theta(1+\alpha)}\right] = \frac{L^{(\alpha+\beta)\theta}}{W^\theta} \times \left[\tau_{ij}^{-\theta}\right] \times \left[\bar{u}_j^{\theta} y_j^{1+\theta} l_j^{\theta(\beta-1)}\right] \iff$$

$$\left[\bar{A}_i^{-\theta} y_i^{1+\theta} l_i^{-\theta(1+\alpha)}\right] = \frac{L^{(\alpha+\beta)\theta}}{W^\theta} \times \left[\left(\mathbf{I}-\mathbf{A_1}\right) - \mathbf{S}\left(\mathbf{I}-\mathbf{A_2}\right)^{-1}\mathbf{S}'\right]^{-1} \times \left[\bar{u}_j^{\theta} y_j^{1+\theta} l_j^{\theta(\beta-1)}\right]$$

where $\left[\bar{A}_i^{-\theta} y_i^{1+\theta} l_i^{-\theta(1+\alpha)}\right]$ and $\left[\bar{u}_j^{\theta} y_j^{1+\theta} l_j^{\theta(\beta-1)}\right]$ are column vectors. Taking a matrix notation and converting back to summation notation:

$$\left[\left(\mathbf{I}-\mathbf{A_1}\right) - \mathbf{S}\left(\mathbf{I}-\mathbf{A_2}\right)^{-1}\mathbf{S}'\right] \times \left[\bar{A}_i^{-\theta} y_i^{1+\theta} l_i^{-\theta(1+\alpha)}\right] = \frac{L^{(\alpha+\beta)\theta}}{W^\theta} \times \left[\bar{u}_i^{\theta} y_i^{1+\theta} l_i^{\theta(\beta-1)}\right] \iff$$

$$\left[\bar{A}_i^{-\theta} y_i^{1+\theta} l_i^{-\theta(1+\alpha)}\right] - \mathbf{A}_1 \times \left[\bar{A}_j^{-\theta} y_j^{1+\theta} l_j^{-\theta(1+\alpha)}\right] - \left[\mathbf{S}\left(\mathbf{I}-\mathbf{A_2}\right)^{-1}\mathbf{S}'\right] \times \left[\bar{A}_j^{-\theta} y_j^{1+\theta} l_j^{-\theta(1+\alpha)}\right] = \frac{L^{(\alpha+\beta)\theta}}{W^\theta} \times \left[\bar{u}_i^{\theta} y_i^{1+\theta} l_i^{\theta(\beta-1)}\right] \iff$$

$$\bar{A}_i^{-\theta} y_i^{1+\theta} l_i^{-\theta(1+\alpha)} - \sum_j a_{ij} \bar{A}_j^{-\theta} y_j^{1+\theta} l_j^{-\theta(1+\alpha)} - \sum_j s_{ii'}^{-\theta} \tau_{i'j'}^{-\theta} s_{j'j}^{-\theta} \bar{A}_j^{-\theta} y_j^{1+\theta} l_j^{-\theta(1+\alpha)} = \frac{L^{(\alpha+\beta)\theta}}{W^\theta} \bar{u}_i^{\theta} y_i^{1+\theta} l_i^{\theta(\beta-1)}$$

where in the last line we used the definition, $\tau_{i'j'}^{-\theta} = [\mathbf{I}-\mathbf{A}_2]_{i'j'}^{-1}$. The second equilibrium condition, equation 9, can also be written as a matrix multiplication, where $\left[\bar{u}_i^{-\theta} y_i^{-\theta} l_i^{\theta(1-\beta)}\right]$ and $\left[\bar{A}_j^{\theta} y_j^{-\theta} l_j^{\theta(\alpha+1)}\right]$ are row vectors. Applying the same matrix inversion, we get:

$$u_i^{-\theta} y_i^{-\theta} l_i^{\theta(1-\beta)} = \frac{\bar{L}^{(\alpha+\beta)\theta}}{W^\theta} \sum_{j=1}^{N} \tau_{ji}^{-\theta} \bar{A}_j^{\theta} y_j^{-\theta} l_j^{\theta(\alpha+1)} \iff$$

$$\left[\tilde{u}_i^{-\theta} y_i^{-\theta} l_i^{\theta(1-\beta)}\right] = \frac{\bar{L}^{(\alpha+\beta)\theta}}{W^\theta} \times \left[\tilde{A}_j^{\theta} y_j^{-\theta} l_j^{\theta(\alpha+1)}\right] \times \left[\tau_{ji}^{-\theta}\right] \iff$$

$$\left[\bar{u}_i^{-\theta} y_i^{-\theta} l_i^{\theta(1-\beta)}\right] = \frac{L^{(\alpha+\beta)\theta}}{W^\theta} \times \left[A_j^{\theta} y_j^{-\theta} l_j^{\theta(\alpha+1)}\right] \times \left[\left(\mathbf{I}-\mathbf{A}_1^T\right) - \mathbf{S}^T\left(\mathbf{I}-\mathbf{A}_2^T\right)^{-1}\mathbf{S}\right]^{-1} \iff$$

$$\left[\bar{u}_i^{-\theta} y_i^{-\theta} l_i^{\theta(1-\beta)}\right] \times \left[\left(\mathbf{I}-\mathbf{A}_1^T\right) - \mathbf{S}^T\left(\mathbf{I}-\mathbf{A}_2^T\right)^{-1}\mathbf{S}\right] = \frac{L^{(\alpha+\beta)\theta}}{W^\theta} \times \left[\bar{A}_i^{\theta} y_i^{-\theta} l_i^{\theta(\alpha+1)}\right] \iff$$

$$\left[\bar{u}_i^{-\theta} y_i^{-\theta} l_i^{\theta(1-\beta)}\right] - \left[\bar{u}_j^{-\theta} y_j^{-\theta} l_j^{\theta(1-\beta)}\right] \times \mathbf{A}_1^T - \left[\bar{u}_j^{-\theta} y_j^{-\theta} l_j^{\theta(1-\beta)}\right] \times \left[\mathbf{S}^T\left(\mathbf{I}-\mathbf{A}_2^T\right)^{-1}\mathbf{S}\right] = \frac{\bar{L}^{(\alpha+\beta)\theta}}{W^\theta} \times \left[\bar{A}_i^{\theta} y_i^{-\theta} l_i^{\theta(\alpha+1)}\right] \iff$$

$$\bar{u}_i^{-\theta} y_i^{-\theta} l_i^{\theta(1-\beta)} - \sum_j a_{ij} \bar{u}_j^{-\theta} y_j^{-\theta} l_j^{\theta(1-\beta)} - \sum_j s_{ii'}^{-\theta} \tau_{i'j'}^{-\theta} s_{j'j}^{-\theta} \bar{u}_j^{-\theta} y_j^{-\theta} l_j^{\theta(1-\beta)} = \frac{L^{(\alpha+\beta)\theta}}{W^\theta} \bar{A}_i^{\theta} y_i^{-\theta} l_i^{\theta(\alpha+1)}$$

Recalling that, $a_{ij} = t_{ij}^{-\theta}$, we have,

$$\bar{A}_i^{-\theta} y_i^{1+\theta} l_i^{-\theta(1+\alpha)} = \frac{L^{(\alpha+\beta)\theta}}{W^\theta} \bar{u}_i^{\theta} y_i^{1+\theta} l_i^{\theta(\beta-1)} + \sum_j t_{ij}^{-\theta} \bar{A}_j^{-\theta} y_j^{1+\theta} l_j^{-\theta(1+\alpha)} + \sum_j s_{ii'}^{-\theta} \tau_{i'j'}^{-\theta} s_{j'j}^{-\theta} \bar{A}_j^{-\theta} y_j^{1+\theta} l_j^{-\theta(1+\alpha)}$$

$$\bar{u}_i^{-\theta} y_i^{-\theta} l_i^{\theta(1-\beta)} = \frac{L^{(\alpha+\beta)\theta}}{W^\theta} \bar{A}_i^{\theta} y_i^{-\theta} l_i^{\theta(\alpha+1)} + \sum_j t_{ji}^{-\theta} \bar{u}_j^{-\theta} y_j^{-\theta} l_j^{\theta(1-\beta)} + \sum_j s_{jj'}^{-\theta} \tau_{j'i'}^{-\theta} s_{i'i}^{-\theta} \bar{u}_j^{-\theta} y_j^{-\theta} l_j^{\theta(1-\beta)}$$

From the section on equlibrium traffic we have,

$$t_{kl} = \bar{t}_{kl}^{\frac{1}{1+\theta\lambda_1}} \times P_k^{-\frac{\theta\lambda_1}{1+\theta\lambda_1}} \times \Pi_l^{-\frac{\theta\lambda_1}{1+\theta\lambda_1}}$$

Converting from price indices,

$$P_i = \frac{1}{\bar{W}} \bar{u}_i L_i^{\beta-1} Y_i$$

$$\Pi_i = \bar{A}_i L_i^{1+\alpha} Y_i^{-\frac{\theta+1}{\theta}}$$

We obtain,

$$\begin{aligned} t_{ij} &= \bar{t}_{ij}^{\frac{1}{1+\theta\lambda_1}} \times (P_i)^{-\frac{\theta\lambda_1}{1+\theta\lambda_1}} \times (\Pi_j)^{-\frac{\theta\lambda_1}{1+\theta\lambda_1}} \\ &= \bar{t}_{ij}^{\frac{1}{1+\theta\lambda}} \times \left(\frac{1}{\bar{W}} \bar{u}_i L_i^{\beta-1} Y_i \right)^{-\frac{\theta\lambda}{1+\theta\lambda}} \times \left(\bar{A}_j L_j^{1+\alpha} Y_j^{-\frac{\theta+1}{\theta}} \right)^{-\frac{\theta\lambda}{1+\theta\lambda}} \end{aligned}$$

Simplifying we obtain,

$$t_{ij}^{-\theta} = \left(\left(\bar{t}_{ij} \bar{L}^{\lambda_1} \right)^{\frac{1}{1+\theta\lambda_1}} \left(\frac{\bar{L}^{-(\alpha+\beta)\theta}}{W^{-\theta}} \right)^{\frac{\lambda_1}{1+\theta\lambda_1}} \bar{A}_j^{-\frac{\theta\lambda_1}{1+\theta\lambda_1}} \bar{u}_i^{-\frac{\theta\lambda_1}{1+\theta\lambda_1}} l_i^{-\frac{\theta\lambda_1(\beta-1)}{1+\theta\lambda_1}} l_j^{-\frac{\theta\lambda_1(1+\alpha)}{1+\theta\lambda_1}} y_i^{-\frac{\theta\lambda_1}{1+\theta\lambda_1}} y_j^{\frac{\lambda_1(1+\theta)}{1+\theta\lambda_1}} \right)^{-\theta}$$

We obtain,

$$\bar{A}_i^{-\theta} y_i^{1+\theta} l_i^{-\theta(1+\alpha)} = \frac{L^{(\alpha+\beta)\theta}}{W^\theta} \bar{u}_i^\theta y_i^{1+\theta} l_i^{\theta(\beta-1)} + \sum_j t_{ij}^{-\theta} \bar{A}_j^{-\theta} y_j^{1+\theta} l_j^{-\theta(1+\alpha)} + \sum_j s_{ii'}^{-\theta} \tau_{i'j'}^{-\theta} s_{j'j}^{-\theta} \bar{A}_j^{-\theta} y_j^{1+\theta} l_j^{-\theta(1+\alpha)}$$

$$\begin{aligned} \bar{A}_i^{-\theta} y_i^{1+\theta} l_i^{-\theta(1+\alpha)} &= \frac{L^{(\alpha+\beta)\theta}}{W^\theta} \bar{u}_i^\theta y_i^{1+\theta} l_i^{\theta(\beta-1)} \\ &+ \sum_j \left(\left(\bar{t}_{ij} \bar{L}^{\lambda_1} \right)^{\frac{1}{1+\theta\lambda_1}} \left(\frac{\bar{L}^{-(\alpha+\beta)\theta}}{W^{-\theta}} \right)^{\frac{\lambda_1}{1+\theta\lambda_1}} \bar{A}_j^{-\frac{\theta\lambda_1}{1+\theta\lambda_1}} \bar{u}_i^{-\frac{\theta\lambda_1}{1+\theta\lambda_1}} l_i^{-\frac{\theta\lambda_1(\beta-1)}{1+\theta\lambda_1}} l_j^{-\frac{\theta\lambda_1(1+\alpha)}{1+\theta\lambda_1}} y_i^{-\frac{\theta\lambda_1}{1+\theta\lambda_1}} y_j^{\frac{\lambda_1(1+\theta)}{1+\theta\lambda_1}} \right)^{-\theta} \\ &\times \bar{A}_j^{-\theta} y_j^{1+\theta} l_j^{-\theta(1+\alpha)} \\ &+ \sum_j s_{ii'}^{-\theta} \tau_{i'j'}^{-\theta} s_{j'j}^{-\theta} \bar{A}_j^{-\theta} y_j^{1+\theta} l_j^{-\theta(1+\alpha)} \end{aligned}$$

$$\begin{aligned} \bar{A}_i^{-\theta} y_i^{1+\theta} l_i^{-\theta(1+\alpha)} &= \frac{L^{(\alpha+\beta)\theta}}{W^\theta} \bar{u}_i^\theta y_i^{1+\theta} l_i^{\theta(\beta-1)} \\ &+ \sum_j \left(\bar{t}_{ij} \bar{L}^{\lambda_1} \right)^{-\frac{\theta}{1+\theta\lambda_1}} \left(\frac{\bar{L}^{(\alpha+\beta)\theta}}{W^{\theta}} \right)^{\frac{\theta\lambda_1}{1+\theta\lambda_1}} \bar{A}_j^{-\frac{\theta}{1+\theta\lambda_1}} \bar{u}_i^{\theta\frac{\theta\lambda_1}{1+\theta\lambda_1}} l_i^{\theta(\beta-1)\frac{\theta\lambda_1}{1+\theta\lambda_1}} y_i^{\theta\frac{\theta\lambda_1}{1+\theta\lambda_1}} y_j^{\frac{1+\theta}{1+\theta\lambda_1}} l_j^{-\frac{\theta(1+\alpha)}{1+\theta\lambda_1}} \\ &+ \sum_j s_{ii'}^{-\theta} \tau_{i'j'}^{-\theta} s_{j'j}^{-\theta} \bar{A}_j^{-\theta} y_j^{1+\theta} l_j^{-\theta(1+\alpha)} \end{aligned}$$

$$
\begin{aligned}
y_i^{\frac{1+\theta\lambda_1+\theta}{1+\theta\lambda_1}} l_i^{\frac{-\theta(1+\alpha+\theta\lambda_1(\alpha+\beta))}{1+\theta\lambda_1}} &= \frac{L^{(\alpha+\beta)\theta}}{W^\theta} \bar{A}_i^\theta \bar{u}_i^\theta y_i^{\frac{1+\theta\lambda_1+\theta}{1+\theta\lambda_1}} l_i^{\frac{\theta(\beta-1)}{1+\theta\lambda_1}} \\
&+ \left(\frac{\bar{L}^{(\alpha+\beta)\theta}}{W^\theta}\right)^{\frac{\theta\lambda_1}{1+\theta\lambda_1}} \sum_j \left(\bar{t}_{ij}\bar{L}^{\lambda_1}\right)^{-\frac{\theta}{1+\theta\lambda_1}} \bar{A}_i^\theta \bar{u}_i^{\theta\frac{\theta\lambda_1}{1+\theta\lambda_1}} \bar{A}_j^{-\frac{\theta}{1+\theta\lambda_1}} y_j^{\frac{1+\theta}{1+\theta\lambda_1}} l_j^{-\frac{\theta(1+\alpha)}{1+\theta\lambda_1}} \\
&+ \left(\bar{A}_i^\theta l_i^{-\theta(\beta-1)\frac{\theta\lambda_1}{1+\theta\lambda_1}} y_i^{-\theta\frac{\theta\lambda_1}{1+\theta\lambda_1}}\right) \sum_j s_{ii'}^{-\theta}\tau_{i'j'}^{-\theta}s_{j'j}^{-\theta}\bar{A}_j^{-\theta} y_j^{1+\theta} l_j^{-\theta(1+\alpha)}
\end{aligned}
$$

$$
\begin{aligned}
y_i^{\frac{1+\theta\lambda_1+\theta}{1+\theta\lambda_1}} l_i^{\frac{-\theta(1+\alpha+\theta\lambda_1(\alpha+\beta))}{1+\theta\lambda_1}} &= \chi \bar{A}_i^\theta \bar{u}_i^\theta y_i^{\frac{1+\theta\lambda_1+\theta}{1+\theta\lambda_1}} l_i^{\frac{\theta(\beta-1)}{1+\theta\lambda_1}} \\
&+ \chi^{\frac{\theta\lambda_1}{1+\theta\lambda_1}} \sum_j \left(\bar{t}_{ij}\bar{L}^{\lambda_1}\right)^{-\frac{\theta}{1+\theta\lambda_1}} \bar{A}_i^\theta \bar{u}_i^{\theta\frac{\theta\lambda_1}{1+\theta\lambda_1}} \bar{A}_j^{-\frac{\theta}{1+\theta\lambda_1}} y_j^{\frac{1+\theta}{1+\theta\lambda_1}} l_j^{-\frac{\theta(1+\alpha)}{1+\theta\lambda_1}} \\
&+ \left(\bar{A}_i^\theta l_i^{-\theta(\beta-1)\frac{\theta\lambda_1}{1+\theta\lambda_1}} y_i^{-\theta\frac{\theta\lambda_1}{1+\theta\lambda_1}}\right) \sum_j s_{ii'}^{-\theta}\tau_{i'j'}^{-\theta}s_{j'j}^{-\theta}\bar{A}_j^{-\theta} y_j^{1+\theta} l_j^{-\theta(1+\alpha)}
\end{aligned}
$$

For the second equilibrium condition,

$$
\bar{u}_i^{-\theta} y_i^{-\theta} l_i^{\theta(1-\beta)} = \frac{L^{(\alpha+\beta)\theta}}{W^\theta} \bar{A}_i^\theta y_i^{-\theta} l_i^{\theta(\alpha+1)} + \sum_j t_{ji}^{-\theta} \bar{u}_j^{-\theta} y_j^{-\theta} l_j^{\theta(1-\beta)} + \sum_j s_{jj'}^{-\theta}\tau_{j'i'}^{-\theta}s_{i'i}^{-\theta}\bar{u}_j^{-\theta} y_j^{-\theta} l_j^{\theta(1-\beta)}
$$

notice that we have,

$$
t_{ji}^{-\theta} = \left(\left(\bar{t}_{ji}\bar{L}^{\lambda_1}\right)^{\frac{1}{1+\theta\lambda_1}} \left(\frac{\bar{L}^{-(\alpha+\beta)\theta}}{W^{-\theta}}\right)^{\frac{\lambda_1}{1+\theta\lambda_1}} \bar{A}_i^{-\frac{\theta\lambda_1}{1+\theta\lambda_1}} \bar{u}_j^{-\frac{\theta\lambda_1}{1+\theta\lambda_1}} l_j^{-\frac{\theta\lambda_1(\beta-1)}{1+\theta\lambda_1}} l_i^{-\frac{\theta\lambda_1(1+\alpha)}{1+\theta\lambda_1}} y_j^{-\frac{\theta\lambda_1}{1+\theta\lambda_1}} y_i^{\frac{\lambda_1(1+\theta)}{1+\theta\lambda_1}}\right)^{-\theta}
$$

substituting,

$$
\begin{aligned}
\bar{u}_i^{-\theta} y_i^{-\theta} l_i^{\theta(1-\beta)} &= \frac{L^{(\alpha+\beta)\theta}}{W^\theta} \bar{A}_i^\theta y_i^{-\theta} l_i^{\theta(\alpha+1)} \\
&+ \sum_j \left(\left(\bar{t}_{ji}\bar{L}^{\lambda_1}\right)^{\frac{1}{1+\theta\lambda_1}} \left(\frac{\bar{L}^{-(\alpha+\beta)\theta}}{W^{-\theta}}\right)^{\frac{\lambda_1}{1+\theta\lambda_1}} \bar{A}_i^{-\frac{\theta\lambda_1}{1+\theta\lambda_1}} \bar{u}_j^{-\frac{\theta\lambda_1}{1+\theta\lambda_1}} l_j^{-\frac{\theta\lambda_1(\beta-1)}{1+\theta\lambda_1}} l_i^{-\frac{\theta\lambda_1(1+\alpha)}{1+\theta\lambda_1}} y_j^{-\frac{\theta\lambda_1}{1+\theta\lambda_1}} y_i^{\frac{\lambda_1(1+\theta)}{1+\theta\lambda_1}}\right)^{-\theta} \\
&\times \bar{u}_j^{-\theta} y_j^{-\theta} l_j^{\theta(1-\beta)} \\
&+ \sum_j s_{jj'}^{-\theta}\tau_{j'i'}^{-\theta}s_{i'i}^{-\theta}\bar{u}_j^{-\theta} y_j^{-\theta} l_j^{\theta(1-\beta)}
\end{aligned}
$$

$$
\begin{aligned}
\bar{u}_i^{-\theta} y_i^{-\theta} l_i^{\theta(1-\beta)} &= \frac{L^{(\alpha+\beta)\theta}}{W^\theta} \bar{A}_i^\theta y_i^{-\theta} l_i^{\theta(\alpha+1)} \\
&+ \sum_j \left(\bar{t}_{ji}\bar{L}^{\lambda_1}\right)^{-\frac{\theta}{1+\theta\lambda_1}} \times \left(\frac{\bar{L}^{(\alpha+\beta)\theta}}{W^\theta}\right)^{\frac{\theta\lambda_1}{1+\theta\lambda_1}} \bar{A}_i^{\theta\frac{\theta\lambda_1}{1+\theta\lambda_1}} \bar{u}_j^{-\frac{\theta}{1+\theta\lambda_1}} l_j^{\frac{\theta(1-\beta)}{1+\theta\lambda_1}} l_i^{\theta(1+\alpha)\frac{\theta\lambda_1}{1+\theta\lambda_1}} y_j^{-\frac{\theta}{1+\theta\lambda_1}} y_i^{-\frac{\theta\lambda_1(1+\theta)}{1+\theta\lambda_1}} \\
&+ \sum_j s_{jj'}^{-\theta}\tau_{j'i'}^{-\theta}s_{i'i}^{-\theta}\bar{u}_j^{-\theta} y_j^{-\theta} l_j^{\theta(1-\beta)}
\end{aligned}
$$

$$
\begin{aligned}
y_i^{-\frac{\theta(1-\lambda_1)}{1+\theta\lambda_1}} l_i^{\frac{\theta(1-\beta-\theta\lambda_1(\alpha+\beta))}{1+\theta\lambda_1}} &= \chi \bar{A}_i^{\theta} \bar{u}_i^{\theta} y_i^{-\frac{\theta(1-\lambda_1)}{1+\theta\lambda_1}} l_i^{\frac{\theta(\alpha+1)}{1+\theta\lambda_1}} \\
&+ \chi^{\frac{\theta\lambda_1}{1+\theta\lambda_1}} \sum_j \left(\bar{t}_{ji} \bar{L}^{\lambda_1}\right)^{-\frac{\theta}{1+\theta\lambda_1}} \bar{A}_i^{\theta\frac{\theta\lambda_1}{1+\theta\lambda_1}} \bar{u}_i^{\theta} \bar{u}_j^{-\frac{\theta}{1+\theta\lambda_1}} l_j^{\frac{\theta(1-\beta)}{1+\theta\lambda_1}} y_j^{-\frac{\theta}{1+\theta\lambda_1}} \\
&+ \left(\bar{u}_i^{\theta} l_i^{-\theta(1+\alpha)\frac{\theta\lambda_1}{1+\theta\lambda_1}} y_i^{\frac{\theta\lambda_1(1+\theta)}{1+\theta\lambda_1}}\right) \sum_j s_{jj'}^{-\theta} \tau_{j'i'}^{-\theta} s_{i'i}^{-\theta} \bar{u}_j^{-\theta} y_j^{-\theta} l_j^{\theta(1-\beta)}
\end{aligned}
$$

In summary we obtain,

$$
\begin{aligned}
y_i^{\frac{1+\theta\lambda_1+\theta}{1+\theta\lambda_1}} l_i^{\frac{-\theta(1+\alpha+\theta\lambda_1(\alpha+\beta))}{1+\theta\lambda_1}} &= \chi \bar{A}_i^{\theta} \bar{u}_i^{\theta} y_i^{\frac{1+\theta\lambda_1+\theta}{1+\theta\lambda_1}} l_i^{\frac{\theta(\beta-1)}{1+\theta\lambda_1}} \\
&+ \chi^{\frac{\theta\lambda_1}{1+\theta\lambda_1}} \sum_j \left(\bar{t}_{ij} \bar{L}^{\lambda_1}\right)^{-\frac{\theta}{1+\theta\lambda_1}} \bar{A}_i^{\theta} \bar{u}_i^{\theta\frac{\theta\lambda_1}{1+\theta\lambda_1}} \bar{A}_j^{-\frac{\theta}{1+\theta\lambda_1}} y_j^{\frac{1+\theta}{1+\theta\lambda_1}} l_j^{-\frac{\theta(1+\alpha)}{1+\theta\lambda_1}} \\
&+ \sum_j s_{ii'}^{-\theta} \tau_{i'j'}^{-\theta} s_{j'j}^{-\theta} \bar{A}_j^{-\theta} y_j^{1+\theta} l_j^{-\theta(1+\alpha)} \bar{A}_i^{\theta} l_i^{-\theta(\beta-1)\frac{\theta\lambda_1}{1+\theta\lambda_1}} y_i^{-\theta\frac{\theta\lambda_1}{1+\theta\lambda_1}}
\end{aligned}
$$

$$
\begin{aligned}
y_i^{-\frac{\theta(1-\lambda_1)}{1+\theta\lambda_1}} l_i^{\frac{\theta(1-\beta-\theta\lambda_1(\alpha+\beta))}{1+\theta\lambda_1}} &= \chi \bar{A}_i^{\theta} \bar{u}_i^{\theta} y_i^{-\frac{\theta(1-\lambda_1)}{1+\theta\lambda_1}} l_i^{\frac{\theta(\alpha+1)}{1+\theta\lambda_1}} \\
&+ \chi^{\frac{\theta\lambda_1}{1+\theta\lambda_1}} \sum_j \left(\bar{t}_{ji} \bar{L}^{\lambda_1}\right)^{-\frac{\theta}{1+\theta\lambda_1}} \bar{A}_i^{\theta\frac{\theta\lambda_1}{1+\theta\lambda_1}} \bar{u}_i^{\theta} \bar{u}_j^{-\frac{\theta}{1+\theta\lambda_1}} l_j^{\frac{\theta(1-\beta)}{1+\theta\lambda_1}} y_j^{-\frac{\theta}{1+\theta\lambda_1}} \\
&+ \sum_j s_{jj'}^{-\theta} \tau_{j'i'}^{-\theta} s_{i'i}^{-\theta} \bar{u}_j^{-\theta} y_j^{-\theta} l_j^{\theta(1-\beta)} \bar{u}_i^{\theta} l_i^{-\theta(1+\alpha)\frac{\theta\lambda_1}{1+\theta\lambda_1}} y_i^{\frac{\theta\lambda_1(1+\theta)}{1+\theta\lambda_1}}
\end{aligned}
$$

Part

Online Appendix

Table of Contents

B Additional Derivations

This section provides additional derivations for the equilibrium equations in Section 3.3, the derivations for the regression design as well as a comparison with Fan and Luo (2020).

B.1 Section 3.3: Equilibrium for Economic Geography Model with Multimodal Routing

Start with the market clearing condition defined as,

$$\begin{aligned} Y_i &= \sum_{j=1}^{N} X_{ij} \iff \\ Y_i &= \sum_{j=1}^{N} \tau_{ij}^{-\theta} w_i^{-\theta} A_i^{\theta} E_j P_j^{\theta} \iff \\ \frac{Y_i}{A_i^{\theta}} w_i^{\theta} &= \sum_{j=1}^{N} \tau_{ij}^{-\theta} E_j P_j^{\theta} \end{aligned}$$

Using overall productivity equation $A_i = \bar{A}_i L_i^{\alpha}$ and total income $Y_i = w_i L_i$

$$\begin{aligned} \frac{Y_i}{A_i^{\theta}} w_i^{\theta} &= \sum_{j=1}^{N} \tau_{ij}^{-\theta} E_j P_j^{\theta} \iff \\ \frac{w_i L_i}{\left(\bar{A}_i L_i^{\alpha}\right)^{\theta}} w_i^{\theta} &= \sum_{j=1}^{N} \tau_{ij}^{-\theta} E_j P_j^{\theta} \\ \bar{A}_i^{-\theta} L_i^{1-\alpha\theta} w_i^{\theta+1} &= \sum_{j=1}^{N} \tau_{ij}^{-\theta} w_j L_j P_j^{\theta} \end{aligned}$$

With welfare equalization where $W_j = \frac{w_j}{P_j} u_j \iff P_j = \frac{w_j}{W_j} u_j$ and overall amenity equation $u_j = \bar{u}_j L_j^{\beta}$ this becomes:

$$\begin{aligned} \bar{A}_i^{-\theta} L_i^{1-\alpha\theta} w_i^{\theta+1} &= \sum_{j=1}^{N} \tau_{ij}^{-\theta} w_j L_j \left(\frac{w_j}{W_j} \bar{u}_j L_j^{\beta}\right)^{\theta} \iff \\ \bar{A}_i^{-\theta} L_i^{1-\alpha\theta} w_i^{\theta+1} &= \sum_{j=1}^{N} \tau_{ij}^{-\theta} w_j L_j w_j^{\theta} \bar{u}_j^{\theta} L_j^{\beta\theta} W^{-\theta} \iff \\ \bar{A}_i^{-\theta} L_i^{1-\alpha\theta} w_i^{\theta+1} &= \sum_{j=1}^{N} \tau_{ij}^{-\theta} w_j^{\theta+1} \bar{u}_j^{\theta} L_j^{\beta\theta+1} W^{-\theta}. \end{aligned}$$

Now defining, share of total income in location i $y_i = \frac{Y_i}{Y^W} = \frac{w_i L_i}{Y^W}$ and share of total labor in location $i l_i = \frac{L_i}{\bar{L}}$

$$
\begin{aligned}
\bar{A}_i^{-\theta} L_i^{1-\alpha\theta} w_i^{\theta+1} &= \sum_{j=1}^{N} \tau_{ij}^{-\theta} w_j^{\theta+1} \bar{u}_j^{\theta} L_j^{\beta\theta+1} W^{-\theta} \iff \\
\bar{A}_i^{-\theta} l_i^{1-\alpha\theta} \bar{L}^{1-\alpha\theta} \left(\frac{y_i Y^W}{l_i \bar{L}}\right)^{\theta+1} &= \sum_{j=1}^{N} \tau_{ij}^{-\theta} \left(\frac{y_i Y^W}{l_i \bar{L}}\right)^{\theta+1} \bar{u}_j^{\theta} L_j^{\beta\theta+1} W^{-\theta} \iff \\
\bar{A}_i^{-\theta} y_i^{\theta+1} l_i^{-\theta(1+\alpha)} \bar{L}^{\theta(1+\alpha)} \left(Y^W\right)^{\theta+1} &= \left(Y^W\right)^{\theta+1} \bar{L}^{\theta(\beta-1)} W^{-\theta} \sum_{j=1}^{N} \tau_{ij}^{-\theta} \bar{u}_j^{\theta} y_j^{1+\theta} l_j^{\theta(\beta-1)} \iff \\
\bar{A}_i^{-\theta} y_i^{1+\theta} l_i^{-\theta(1+\alpha)} &= \frac{\bar{L}^{(\alpha+\beta)\theta}}{W^{\theta}} \sum_{j=1}^{N} \tau_{ij}^{-\theta} \bar{u}_j^{\theta} y_j^{1+\theta} l_j^{\theta(\beta-1)} \\
&= \chi \sum_{j=1}^{N} \tau_{ij}^{-\theta} \bar{u}_j^{\theta} y_j^{1+\theta} l_j^{\theta(\beta-1)}
\end{aligned}
$$

Similarly for the balanced trade condition,

$$
\begin{aligned}
E_i &= \sum_{j=1}^{N} X_{ji} \iff \\
E_i &= \sum_{j=1}^{N} \tau_{ji}^{-\theta} w_j^{-\theta} A_j^{\theta} E_i P_i^{\theta} \iff \\
P_i^{-\theta} &= \sum_{j=1}^{N} \tau_{ji}^{-\theta} w_j^{-\theta} \widetilde{A}_j^{\theta} L_j^{\alpha\theta}.
\end{aligned}
$$

Assuming welfare equalization,

$$
W^{\theta} w_i^{-\theta} \bar{u}_i^{-\theta} L_i^{-\beta\theta} = \sum_{j=1}^{N} \tau_{ji}^{-\theta} w_j^{-\theta} \bar{A}_j^{\theta} L_j^{\alpha\theta}.
$$

Defining income and labor shares as above,

$$
\begin{aligned}
W^{\theta} w_i^{-\theta} \bar{u}_i^{-\theta} L_i^{-\beta\theta} &= \sum_{j=1}^{N} \tau_{ji}^{-\theta} w_j^{-\theta} \bar{A}_j^{\theta} L_j^{\alpha\theta} \iff \\
W^{\theta} \left(\frac{y_i Y^W}{l_i \bar{L}}\right)^{-\theta} \bar{u}_i^{-\theta} l_i^{-\beta\theta} \bar{L}^{-\beta\theta} &= \sum_{j=1}^{N} \tau_{ji}^{-\theta} \left(\frac{y_j Y^W}{l_j \bar{L}}\right)^{-\theta} \bar{A}_j^{\theta} l_j^{\alpha\theta} \bar{L}^{\alpha\theta} \iff \\
W^{\theta} y_i^{-\theta} l_i^{\theta(1-\beta)} \bar{u}_i^{-\theta} \left(Y^W\right)^{-\theta} \bar{L}^{-\beta\theta} &= \left(Y^W\right)^{-\theta} \bar{L}^{\theta(\alpha+1)} \sum_{j=1}^{N} \tau_{ji}^{-\theta} y_j^{-\theta} l_j^{\theta(\alpha+1)} \bar{A}_j^{\theta} \iff \\
\bar{u}_i^{-\theta} y_i^{-\theta} l_i^{\theta(1-\beta)} &= \frac{\bar{L}^{(\alpha+\beta)\theta}}{W^{\theta}} \sum_{j=1}^{N} \tau_{ji}^{-\theta} \bar{A}_j^{\theta} y_j^{-\theta} l_j^{\theta(\alpha+1)}
\end{aligned}
$$

B.2 Regression design for modal diversion

We are interested in deriving a regression that studies the impact of secondary network traffic in a specific MSA with regard to plausibly exogenous changes in the primary network (road) transportation cost in the same location. Consider an MSA located at node k. We will make an additional assumption that there exists some localized primary network fully contained within the MSA that any unimodal route originating or terminating in k needs to transition through before accessing the national primary road network. Let this localized network be represented by the transportation cost $\bar{t}_k$. We are therefore interested in running the following regression,

$$d\ln \Xi_{kk'} = \alpha + \beta_k \times d\ln \bar{t}_k + \epsilon_{kk'}$$

where $\Xi_{kk'}$ refers to the amount of traffic at the intermodal station in k and represents traffic from the primary to the secondary network in that location and $d\ln \bar{t}_k$ refers to changes in transportation cost of the localized primary network.

Given the assumption above we can simplify the expression for the unimodal transportation cost (Equation XZ), i.e.

$$\begin{aligned}\left(\tau_{kj}^{1}\right)^{-\theta} &= \left(\sum_{r\in\mathcal{R}_{ij}^{1}}\left(\prod_{l=1}^{K} t_{r_{l-1},r_l}^{-\theta}\right)\right)\\ &= t_k^{-\theta}\left(\sum_{r\in\mathcal{R}_{ij}^{1}}\left(\prod_{l=2}^{K} t_{r_{l-1},r_l}^{-\theta}\right)\right)\end{aligned}$$

which factorizes out the transportation cost associated with the localized network, $t_k^{-\theta}$, since it is assumed to be used on all routes.

In order to derive the regression we start from equilibrium traffic at terminal stations (Equation XZ), i.e.

$$\Xi_{kk'}^{2} = \bar{s}_{kk'}^{-\frac{\theta}{1+\theta\lambda_2}} \times P_k^{-\frac{\theta}{1+\theta\lambda_2}} \times \left(\sum_{l} \tau_{k'l'}^{-\theta} s_{l'l}^{-\theta} \Pi_l^{-\theta}\right)^{\frac{1}{1+\theta\lambda_2}}$$

and we examine the responsiveness of traffic flows with regard to changes in the price index, $d\ln P_k$, which implies,

$$d\ln \Xi_{kk'} = d\ln P_k^{-\frac{\theta}{1+\theta\lambda_2}}$$

we then differentiate the price index and examine the responsiveness of the price index with regard to changes in the transportation cost,

$$P_k^{-\frac{\theta}{1+\theta\lambda_2}} = \left(\sum_{i=1}^{N} \tau_{ik}^{-\theta} Y_i \Pi_i^{\theta}\right)^{\frac{1}{1+\theta\lambda_2}}$$

$$dP_k^{-\frac{\theta}{1+\theta\lambda_2}} = \frac{1}{1+\theta\lambda_2} \times \left(\sum_{i=1}^{N} Y_i \Pi_i^{\theta} \tau_{ik}^{-\theta}\right)^{\frac{1}{1+\theta\lambda_2}-1} \times \sum_{i} Y_i \Pi_i^{\theta} d\tau_{ik}^{-\theta}$$

$$d\ln P_k^{-\frac{\theta}{1+\theta\lambda_2}} = \frac{1}{1+\theta\lambda_2} \sum_{i=1}^{N} \frac{\tau_{ik}^{-\theta} Y_i \Pi_i^{\theta}}{\sum_{i=1}^{N} \tau_{ik}^{-\theta} Y_i \Pi_i^{\theta}} d\ln \tau_{ik}^{-\theta}$$

Instead of considering arbitrary changes to the primary road transportation network instead focus on changes in the transportation cost at node k only. Totally differentiating the expression for unimodal transportation costs (Equation XZ), we obtain,

$$d\ln\left(\tau_{kj}^{1}\right)^{-\theta} = -\theta d\ln t_k \forall j$$

Combining we obtain,

$$\begin{aligned} d\ln P_k^{-\frac{\theta}{1+\theta\lambda_2}} &= -\frac{\theta}{1+\theta\lambda_2} d\ln t_k \sum_{i=1}^{N} \frac{\tau_{ik}^{-\theta} Y_i \Pi_i^{\theta}}{\sum_{i=1}^{N} \tau_{ik}^{-\theta} Y_i \Pi_i^{\theta}} d\ln \tau_{ik}^{-\theta} \\ &= -\frac{\theta}{1+\theta\lambda_2} d\ln t_k \end{aligned}$$

we have,

$$d\ln \Xi_{kk'} = d\ln P_k^{-\frac{\theta}{1+\theta\lambda_2}}$$

combining we have,

$$d\ln \Xi_{kk'} = -\frac{\theta}{1+\theta\lambda_2} d\ln t_{k,u}$$

Furthermore, we have,

$$t_{kl} = \bar{t}_{kl}^{\frac{1}{1+\theta\lambda_1}} \times P_k^{-\frac{\theta\lambda_1}{1+\theta\lambda_1}} \times \Pi_l^{-\frac{\theta\lambda_1}{1+\theta\lambda_1}}$$

which implies,

$$d\ln t_{kl} = \frac{1}{1+\theta\lambda_1} d\ln \bar{t}_{kl}$$

plugging in, we finally obtain,

$$d\ln \Xi_{kk'} = -\theta \frac{1}{1+\theta\lambda_2} \frac{1}{1+\theta\lambda_1} d\ln \bar{t}_{k,u}$$

which implies that the structural elasticity depends on the separate congestion forces on the primary and secondary network as well as the strength of substitution between routes and modes.

B.3 Comparison with Fan and Luo (2020)

Our project studies multimodal transport networks and their impact on the returns to new technology and infrastructure investment. In particular, we focus on how these outcomes depend on the geography of the multimodal network as well as the intermodal terminals which allows for switches between transport modes. To this end, we develop a quantitative spatial equilibrium model by extending the routing-based formulation of transport cost in Allen and Arkolakis (2022) to incorporate transportation across multiple transport modes and possible mode switching conditional on the geography of the multimodal transport network, including the location of intermodal terminals, and incurred switching costs. Employing the properties of partitioned matrices, we derive closed-form expressions for the expected transport cost over the multimodal transport network despite the increased dimensionality and complexity of the underlying network. This tractable model of routing allows for counterfactual experiments which allows us to evaluate the welfare consequences to new technology and modal or terminal infrastructure improvements.

In what follows, we demonstrate that our results are consistent with the transportation cost derived in Proposition 1 in Fan and Luo (2020). Fan and Luo (2020) is a note which presents a model of transshipment building on Allen and Arkolakis (2022) and Fan, Lu and Luo (2019).

We first restate our expected transport cost from origin i to destination j over the multimodal transportation network below for convenience (equation (16) from our theory section):

$$\tau_{ij} = e_{ij}^{-\frac{1}{\theta}}$$

where $\mathbf{E} = [e_{ij}]$ refers to the inverse of the Schur complement of the Laplacian of the partitioned infrastructure matrix for the multimodal transport network and is defined as follows,

$$\mathbf{E} \equiv \left(\mathbf{B}^{-1} - \mathbf{D}\right)^{-1} \equiv S(\Omega)^{-1}$$

where $\mathbf{B} \equiv (\mathbf{I} - \mathbf{A}_1)^{-1}$ is the geometric sum of the primary transport network matrix $\mathbf{A}_1$ and $\mathbf{D} \equiv \mathbf{S}\left(\sum_{K=0}^{\infty} \mathbf{A}_2^K\right)\mathbf{S}'$ is the geometric sum of the secondary transport matrix $\mathbf{A}_2$ that is inclusive of intermodal switching linkages between the primary and secondary transport network $\mathbf{S}$.

To do so, instead of deriving the matrix representation from the explicit numeration and recursive formula as in the main text, we instead employ the inverse of the Leontief matrix of the underlying infrastructure matrix

$$\begin{aligned}
\tau_{ij} &\equiv \lim_{N\to\infty} \tau_{ij,N} \\
&= \lim_{N\to\infty} \Gamma\left(\frac{\theta-1}{\theta}\right)\left(\sum_{K=1}^{N}\left[\Omega^K_{(i,j)}\right]\right)^{-\frac{1}{\theta}} \\
&= \Gamma\left(\frac{\theta-1}{\theta}\right)\left(\left[(\mathbf{I}-\mathbf{\Omega})^{-1}_{(i,j)}\right]\right)^{-\frac{1}{\theta}}, i \neq j \\
&= \Gamma\left(\frac{\theta-1}{\theta}\right)\left(\begin{bmatrix} \mathbf{B} + \mathbf{BS}\left(S(\Omega)^{-1}\right)\mathbf{S}'\mathbf{B} & \mathbf{BS}\left(S(\Omega)^{-1}\right) \\ \left(S(\Omega)^{-1}\right)\mathbf{SB} & \left(S(\Omega)^{-1}\right) \end{bmatrix}_{(i,j)}\right)^{-\frac{1}{\theta}}
\end{aligned}$$

where N is maximum number of edges each ij pair can have, and $S(\Omega)$ defines the Schur complement of the Laplacian of the partitioned infrastructure matrix, as above.

If the freight shipment originates and terminates on the primary network, commonly known as the first and last mile in freight transportation, we can then express bilateral transportation costs more succinctly below as a decomposition of the transport costs that arise from (A) the universe of unimodal transportation over the primary network, and (B) the additional multimodal transportation over the primary network that traverses through the secondary network taking into account the possible linkages between the two networks:

$$\tau_{ij}^{-\theta} = \left[\underbrace{\mathbf{B}}_{\substack{\text{Unimodal costs over} \\ \text{primary network}}} + \underbrace{\mathbf{BS}\left(S(\Omega)^{-1}\right)\mathbf{S}'\mathbf{B}}_{\substack{\text{Multimodal costs over} \\ \text{primary \& secondary networks}}} \right]_{ij} = \left(\tau_{ij}^{1}\right)^{-\theta} + \left(\tau_{ij}^{1,2}\right)^{-\theta} \tag{44}$$

which corresponds to equation (17) in our draft and is an alternative expression that is equivalent to equation (16). The first and second terms in the equation above corresponds to items (A) and (B) respectively.

If we abstract from this first and last mile assumption—that freight shipments originate and terminate on the primary network—then we can trace out the sum of paths that might either originate or terminate on either network. In matrix notation this corresponds to the following:

$$\begin{aligned} \tau_{ij} &= \Gamma\left(\frac{\theta-1}{\theta}\right) \cdot \left(\left[[\mathbf{A}_1 \mathbf{A}_2] \begin{bmatrix} \mathbf{B} + \mathbf{BS}\left(S(\Omega)^{-1}\right)\mathbf{S}'\mathbf{B} & \mathbf{BS}\left(S(\Omega)^{-1}\right) \\ \left(S(\Omega)^{-1}\right)\mathbf{SB} & \left(S(\Omega)^{-1}\right) \end{bmatrix} \begin{bmatrix} \mathbf{I} \\ \mathbf{I} \end{bmatrix} \right]_{ij} \right)^{-\frac{1}{\theta}} \\ &= \Gamma\left(\frac{\theta-1}{\theta}\right) \cdot \left(\left[[\mathbf{A}_1 \mathbf{A}_2] \left(\mathbf{I} - \mathbf{\Omega}\right)^{-1} \begin{bmatrix} \mathbf{I} \\ \mathbf{I} \end{bmatrix} \right]_{ij} \right)^{-\frac{1}{\theta}} \end{aligned} \tag{45}$$

where the second line above corresponds to the expression for transportation costs in Proposition 1 Part (a) of Fan and Luo (2020), but where the first line utilizes the block matrix structure to make explicit the underlying decomposition that we are introducing in our framework.

C Proofs

This section presents the proof for Proposition 1.

C.1 Proof of Proposition 1: Counterfactual Equilibrium

We proceed in two steps. In a first step we derive the change in the equilibrium conditions in terms of market access terms before then substitution the model specific elements.

C.1.1 Preliminaries

We can write equilibrium trade flows as,

$$X_{ij} = \tau_{ij}^{-\theta} \times \frac{\gamma_i}{\Pi_i^{-\theta}} \times \frac{\delta_j}{P_j^{-\theta}}$$

where γ_i and δ_i are comulative flows out of an origin and into a destination, respectively, and Π_i and P_j are origin and destination market access terms. Given the multimodal routing formulation, trade costs can be represented as:

$$\tau_{ij}^{-\theta} = \left[(\mathbf{I} - \mathbf{A_1}) - \mathbf{S} \left(\mathbf{I} - \mathbf{A_2} \right)^{-1} \mathbf{S'} \right]_{ij}^{-1}$$

For both models we have,

$$\gamma_i = \sum_j X_{ij}$$

$$\delta_i = \sum_j X_{ji}$$

Starting with the first equilibrium condition:

$$\begin{aligned}
\gamma_i &= \sum_j X_{ij} \iff \\
\gamma_i &= \sum_j \tau_{ij}^{-\theta} \times \frac{\gamma_i}{\Pi_i^{-\theta}} \times \frac{\delta_j}{P_j^{-\theta}} \iff \\
\gamma_i &= \sum_j \left[(\mathbf{I} - \mathbf{A_1}) - \mathbf{S} \left(\mathbf{I} - \mathbf{A_2} \right)^{-1} \mathbf{S'} \right]_{ij}^{-1} \times \frac{\gamma_i}{\Pi_i^{-\theta}} \times \frac{\delta_j}{P_j^{-\theta}} \iff \\
\Pi_i^{-\theta} &= \sum_j \left[(\mathbf{I} - \mathbf{A_1}) - \mathbf{S} \left(\mathbf{I} - \mathbf{A_2} \right)^{-1} \mathbf{S'} \right]_{ij}^{-1} \times \frac{\delta_j}{P_j^{-\theta}} \iff \\
\left((\mathbf{I} - \mathbf{A_1}) - \mathbf{S} \left(\mathbf{I} - \mathbf{A_2} \right)^{-1} \mathbf{S'} \right) \Pi_i^{-\theta} &= \frac{\delta_i}{P_i^{-\theta}} \iff \\
\Pi_i^{-\theta} &= \frac{\delta_i}{P_i^{-\theta}} + \sum_j t_{ij}^{-\theta} \Pi_j^{-\theta} + \sum_j s_{ii'}^{-\theta} \tau_{i'j'}^{-\theta} s_{j'j}^{-\theta} \Pi_j^{-\theta}
\end{aligned}$$

Continuing with the second equilibrium condition,

$$
\begin{aligned}
\delta_i &= \sum_j X_{ji} \iff \\
\delta_j &= \sum_j \tau_{ji}^{-\theta} \times \frac{\gamma_j}{\Pi_j^{-\theta}} \times \frac{\delta_i}{P_i^{-\theta}} \iff \\
P_i^{-\theta} &= \sum_j \tau_{ji}^{-\theta} \times \frac{\gamma_j}{\Pi_j^{-\theta}} \iff \\
P_i^{-\theta} &= \sum_j \left[(\mathbf{I}-\mathbf{A_1}) - \mathbf{S}\left(\mathbf{I}-\mathbf{A_2}\right)^{-1}\mathbf{S}'\right]_{ji}^{-1} \times \frac{\gamma_j}{\Pi_j^{-\theta}} \iff \\
\left((\mathbf{I}-\mathbf{A_1}) - \mathbf{S}\left(\mathbf{I}-\mathbf{A_2}\right)^{-1}\mathbf{S}'\right) P_i^{-\theta} &= \frac{\gamma_i}{\Pi_i^{-\theta}} \iff \\
P_i^{-\theta} &= \frac{\gamma_i}{\Pi_i^{-\theta}} + \sum_j t_{ji}^{-\theta} P_j^{-\theta} + \sum_j s_{jj'}^{-\theta}\tau_{j'i'}^{-\theta}s_{i'i}^{-\theta}P_j^{-\theta}
\end{aligned}
$$

Expressed in changes,

$$
\begin{aligned}
\hat{\Pi}_i^{-\theta} = &\left(\frac{\frac{\delta_i}{P_i^{-\theta}}}{\frac{\delta_i}{P_i^{-\theta}} + \sum_j t_{ij}^{-\theta}\Pi_j^{-\theta} + \sum_j s_{ii'}^{-\theta}\tau_{i'j'}^{-\theta}s_{j'j}^{-\theta}\Pi_j^{-\theta}}\right)\frac{\hat{\delta}_i}{\hat{P}_i^{-\theta}} \\
&+ \sum_j \left(\frac{t_{ij}^{-\theta}\Pi_j^{-\theta}}{\frac{\delta_i}{P_i^{-\theta}} + \sum_j t_{ij}^{-\theta}\Pi_j^{-\theta} + \sum_j s_{ii'}^{-\theta}\tau_{i'j'}^{-\theta}s_{j'j}^{-\theta}\Pi_j^{-\theta}}\right)\hat{t}_{ij}^{-\theta}\hat{\Pi}_j^{-\theta} \\
&+ \sum_j \left(\frac{s_{ii'}^{-\theta}\tau_{i'j'}^{-\theta}s_{j'j}^{-\theta}\Pi_j^{-\theta}}{\frac{\delta_i}{P_i^{-\theta}} + \sum_j t_{ij}^{-\theta}\Pi_j^{-\theta} + \sum_j s_{ii'}^{-\theta}\tau_{i'j'}^{-\theta}s_{j'j}^{-\theta}\Pi_j^{-\theta}}\right)\hat{s}_{ii'}^{-\theta}\hat{\tau}_{i'j'}^{-\theta}\hat{s}_{j'j}^{-\theta}\hat{\Pi}_j^{-\theta}
\end{aligned}
$$

and:

$$
\begin{aligned}
\hat{P}_i^{-\theta} = &\left(\frac{\frac{\gamma_i}{\Pi_i^{-\theta}}}{\frac{\gamma_i}{\Pi_i^{-\theta}} + \sum_j t_{ji}^{-\theta}P_j^{-\theta} + \sum_j s_{ii'}^{-\theta}\tau_{j'i'}^{-\theta}s_{j'j}^{-\theta}P_j^{-\theta}}\right)\frac{\hat{\gamma}_i}{\hat{\Pi}_i^{-\theta}} \\
&+ \sum_j \left(\frac{t_{ji}^{-\theta}P_j^{-\theta}}{\frac{\gamma_i}{\Pi_i^{-\theta}} + \sum_j t_{ji}^{-\theta}P_j^{-\theta} + \sum_j s_{ii'}^{-\theta}\tau_{j'i'}^{-\theta}s_{j'j}^{-\theta}P_j^{-\theta}}\right)\hat{t}_{ji}^{-\theta}\hat{P}_j^{-\theta} \\
&+ \sum_j \left(\frac{s_{ii'}^{-\theta}\tau_{j'i'}^{-\theta}s_{j'j}^{-\theta}P_j^{-\theta}}{\frac{\gamma_i}{\Pi_i^{-\theta}} + \sum_j t_{ji}^{-\theta}P_j^{-\theta} + \sum_j s_{ii'}^{-\theta}\tau_{j'i'}^{-\theta}s_{j'j}^{-\theta}P_j^{-\theta}}\right)\hat{s}_{ii'}^{-\theta}\hat{\tau}_{j'i'}^{-\theta}\hat{s}_{j'j}^{-\theta}\hat{P}_j^{-\theta}
\end{aligned}
$$

Multiplying both the numerator and denominator by their appropriate market acccess term,

$$
\begin{aligned}
\hat{\Pi}_i^{-\theta} = &\left(\frac{\delta_i}{\delta_i + \sum_j t_{ij}^{-\theta}\Pi_j^{-\theta}P_i^{-\theta} + \sum_j s_{ii'}^{-\theta}\tau_{i'j'}^{-\theta}s_{j'j}^{-\theta}\Pi_j^{-\theta}P_i^{-\theta}}\right)\frac{\hat{\delta}_i}{\hat{P}_i^{-\theta}} \\
&+ \sum_j \left(\frac{t_{ij}^{-\theta}\Pi_j^{-\theta}P_i^{-\theta}}{\delta_i + \sum_j t_{ij}^{-\theta}\Pi_j^{-\theta}P_i^{-\theta} + \sum_j s_{ii'}^{-\theta}\tau_{i'j'}^{-\theta}s_{j'j}^{-\theta}\Pi_j^{-\theta}P_i^{-\theta}}\right)\hat{t}_{ij}^{-\theta}\hat{\Pi}_j^{-\theta} \\
&+ \sum_j \left(\frac{s_{ii'}^{-\theta}\tau_{i'j'}^{-\theta}s_{j'j}^{-\theta}\Pi_j^{-\theta}P_i^{-\theta}}{\delta_i + \sum_j t_{ij}^{-\theta}\Pi_j^{-\theta}P_i^{-\theta} + \sum_j s_{ii'}^{-\theta}\tau_{i'j'}^{-\theta}s_{j'j}^{-\theta}\Pi_j^{-\theta}P_i^{-\theta}}\right)\hat{s}_{ii'}^{-\theta}\hat{\tau}_{i'j'}^{-\theta}\hat{s}_{j'j}^{-\theta}\hat{\Pi}_j^{-\theta}
\end{aligned}
$$

$$
\begin{aligned}
\hat{\Pi}_i^{-\theta} = &\left(\frac{\delta_i}{\delta_i + \sum_j \Xi^1_{ij} + \sum_j \Xi^2_{ij}}\right)\frac{\hat{\delta}_i}{\hat{P}_i^{-\theta}} + \sum_j \left(\frac{\Xi^1_{ij}}{\delta_i + \sum_j \Xi^1_{ij} + \sum_j \Xi^2_{ij}}\right)\hat{t}_{ij}^{-\theta}\hat{\Pi}_j^{-\theta} \\
&+ \sum_j \left(\frac{\Xi^2_{ij}}{\delta_i + \sum_j \Xi^1_{ij} + \sum_j \Xi^2_{ij}}\right)\hat{s}_{ii'}^{-\theta}\hat{\tau}_{i'j'}^{-\theta}\hat{s}_{j'j}^{-\theta}\hat{\Pi}_j^{-\theta}
\end{aligned}
$$

and:

$$
\begin{aligned}
\hat{P}_i^{-\theta} = &\left(\frac{\gamma_i}{\gamma_i + \sum_j t_{ji}^{-\theta}P_j^{-\theta}\Pi_i^{-\theta} + \sum_j s_{ii'}^{-\theta}\tau_{j'i'}^{-\theta}s_{j'j}^{-\theta}P_j^{-\theta}\Pi_i^{-\theta}}\right)\frac{\hat{\gamma}_i}{\hat{\Pi}_i^{-\theta}} \\
&+ \sum_j \left(\frac{t_{ji}^{-\theta}P_j^{-\theta}\Pi_i^{-\theta}}{\gamma_i + \sum_j t_{ji}^{-\theta}P_j^{-\theta}\Pi_i^{-\theta} + \sum_j s_{ii'}^{-\theta}\tau_{j'i'}^{-\theta}s_{j'j}^{-\theta}P_j^{-\theta}\Pi_i^{-\theta}}\right)\hat{t}_{ji}^{-\theta}\hat{P}_j^{-\theta} \\
&+ \sum_j \left(\frac{s_{ii'}^{-\theta}\tau_{j'i'}^{-\theta}s_{j'j}^{-\theta}P_j^{-\theta}\Pi_i^{-\theta}}{\gamma_i + \sum_j t_{ji}^{-\theta}P_j^{-\theta}\Pi_i^{-\theta} + \sum_j s_{ii'}^{-\theta}\tau_{j'i'}^{-\theta}s_{j'j}^{-\theta}P_j^{-\theta}\Pi_i^{-\theta}}\right)\hat{s}_{ii'}^{-\theta}\hat{\tau}_{j'i'}^{-\theta}\hat{s}_{j'j}^{-\theta}\hat{P}_j^{-\theta}
\end{aligned}
$$

$$
\begin{aligned}
\hat{P}_i^{-\theta} = &\left(\frac{\gamma_i}{\gamma_i + \sum_j \Xi^1_{ji} + \sum_j \Xi^2_{ji}}\right)\frac{\hat{\gamma}_i}{\hat{\Pi}_i^{-\theta}} + \sum_j \left(\frac{\Xi^1_{ji}}{\gamma_i + \sum_j \Xi^1_{ji} + \sum_j \Xi^2_{ji}}\right)\hat{t}_{ji}^{-\theta}\hat{P}_j^{-\theta} \\
&+ \sum_j \left(\frac{\Xi^2_{ji}}{\gamma_i + \sum_j \Xi^1_{ji} + \sum_j \Xi^2_{ji}}\right)\hat{s}_{ii'}^{-\theta}\hat{\tau}_{j'i'}^{-\theta}\hat{s}_{j'j}^{-\theta}\hat{P}_j^{-\theta}
\end{aligned}
$$

Finally deriving the change of equilibrium switching costs,

$$
s_{k'k} = \bar{s}_{k'k}^{\frac{1}{1+\theta\lambda_2}} \times \Pi_k^{-\frac{\theta\lambda_2}{1+\theta\lambda_2}} \times \left(\sum_l s_{ll'}^{-\theta}\tau_{l'k'}^{-\theta}P_l^{-\theta}\right)^{\frac{\lambda_2}{1+\theta\lambda_2}}
$$

$$
s_{kk'} = \bar{s}_{kk'}^{\frac{1}{1+\theta\lambda_2}} \times P_k^{-\frac{\theta\lambda_2}{1+\theta\lambda_2}} \times \left(\sum_l \tau_{k'l'}^{-\theta}s_{l'l}^{-\theta}\Pi_l^{-\theta}\right)^{\frac{\lambda_2}{1+\theta\lambda_2}}
$$

Forming time ratios,

$$
\begin{aligned}
\sum_l \widehat{s_{ll'}^{-\theta}\tau_{l'k'}^{-\theta}}P_l^{-\theta} &= \sum_l \frac{\tau_{l'k'}^{-\theta}s_{l'l}^{-\theta}P_l^{-\theta}}{\sum_l \tau_{l'k'}^{-\theta}s_{l'l}^{-\theta}P_l^{-\theta}}\hat{\tau}_{l'k'}^{-\theta}\hat{s}_{ll'}^{-\theta}\hat{P}_l^{-\theta} \\
&= \sum_l \frac{\Xi^2_{l'k'}}{\sum_{l'} \Xi^2_{l'k'}}\hat{\tau}_{l'k'}^{-\theta}\hat{s}_{ll'}^{-\theta}\hat{P}_l^{-\theta}
\end{aligned}
$$

Similarly,

$$
\begin{aligned}
\sum_l \widehat{\tau_{k'l'}^{-\theta}s_{l'l}^{-\theta}}\Pi_l^{-\theta} &= \sum_l \frac{\tau_{k'l'}^{-\theta}s_{l'l}^{-\theta}\Pi_l^{-\theta}}{\sum_l \tau_{k'l'}^{-\theta}s_{l'l}^{-\theta}\Pi_l^{-\theta}}\hat{\tau}_{k'l'}^{-\theta}\hat{s}_{l'l}^{-\theta}\hat{\Pi}_l^{-\theta} \\
&= \sum_l \frac{\Xi^2_{k'l'}}{\sum_{l'} \Xi^2_{k'l'}}\hat{\tau}_{k'l'}^{-\theta}\hat{s}_{l'l}^{-\theta}\hat{\Pi}_l^{-\theta}
\end{aligned}
$$

where in the second line we multiplier in both the nominator and denominator by the appropriate market access terms and switching costs $s_{k'l'}$ and substitute with flows along the secondary network. We obtain the expression for changes in the equilibrium switching costs,

$$
\hat{s}_{k'k} = \hat{\bar{s}}_{k'k}^{\frac{1}{1+\theta\lambda_2}} \times \hat{\Pi}_k^{-\frac{\theta\lambda_2}{1+\theta\lambda_2}} \times \left(\sum_l \frac{\Xi^2_{l'k'}}{\sum_{l'} \Xi^2_{l'k'}}\hat{\tau}_{l'k'}^{-\theta}\hat{s}_{ll'}^{-\theta}\hat{P}_l^{-\theta}\right)^{\frac{\lambda_2}{1+\theta\lambda_2}}
$$

$$\hat{s}_{kk'} = \hat{\bar{s}}_{kk'}^{\frac{1}{1+\theta\lambda_2}} \times \hat{P}_k^{-\frac{\theta\lambda_2}{1+\theta\lambda_2}} \times \left(\sum_l \frac{\Xi^2_{k'l'}}{\sum_{l'} \Xi^2_{k'l'}} \hat{\tau}_{k'l'}^{-\theta} \hat{s}_{l'l}^{-\theta} \hat{\Pi}_l^{-\theta}\right)^{\frac{\lambda_2}{1+\theta\lambda_2}}$$

notice that depending on whether the transport cost is backward or forward oriented different market access terms matter. Substituting in the expression for iceberg trade costs along a link,

$$\hat{t}_{ij} = \hat{\bar{t}}_{ij}^{\frac{1}{1+\theta\lambda_1}} \times \hat{P}_i^{-\frac{\theta\lambda_1}{1+\theta\lambda_1}} \times \hat{\Pi}_j^{-\frac{\theta\lambda_1}{1+\theta\lambda_1}}$$

substituting,

$$\begin{aligned}
\hat{\Pi}_i^{-\theta} = &\left(\frac{\delta_i}{\delta_i + \sum_j \Xi^1_{ij} + \sum_j \Xi^2_{ij}}\right) \frac{\hat{\delta}_i}{\hat{P}_i^{-\theta}} + \sum_j \left(\frac{\Xi^1_{ij}}{\delta_i + \sum_j \Xi^1_{ij} + \sum_j \Xi^2_{ij}}\right) \hat{t}_{ij}^{-\theta} \hat{\Pi}_j^{-\theta} \\
&+ \sum_j \left(\frac{\Xi^2_{ij}}{\delta_i + \sum_j \Xi^1_{ij} + \sum_j \Xi^2_{ij}}\right) \hat{s}_{ii'}^{-\theta} \hat{\tau}_{i'j'}^{-\theta} \hat{s}_{j'j}^{-\theta} \hat{\Pi}_j^{-\theta}
\end{aligned}$$

$$\begin{aligned}
\hat{P}_i^{-\theta} = &\left(\frac{\gamma_i}{\gamma_i + \sum_j \Xi^1_{ji} + \sum_j \Xi^2_{ji}}\right) \frac{\hat{\gamma}_i}{\hat{\Pi}_i^{-\theta}} + \sum_j \left(\frac{\Xi^1_{ij}}{\gamma_i + \sum_j \Xi^1_{ji} + \sum_j \Xi^2_{ji}}\right) \hat{t}_{ji}^{-\theta} \hat{P}_j^{-\theta} \\
&+ \sum_j \left(\frac{\Xi^2_{ij}}{\gamma_i + \sum_j \Xi^1_{ji} + \sum_j \Xi^2_{ji}}\right) \hat{s}_{jj'}^{-\theta} \hat{\tau}_{j'i'}^{-\theta} \hat{s}_{i'i}^{-\theta} \hat{P}_j^{-\theta}
\end{aligned}$$

and multiplying each equation by the other market access term,

$$\begin{aligned}
\hat{\Pi}_i^{-\theta} \hat{P}_i^{-\theta} = &\left(\frac{\delta_i}{\delta_i + \sum_j \Xi^1_{ij} + \sum_j \Xi^2_{ij}}\right) \hat{\delta}_i + \sum_j \left(\frac{\Xi^1_{ij}}{\delta_i + \sum_j \Xi^1_{ij} + \sum_j \Xi^2_{ij}}\right) \hat{t}_{ij}^{-\frac{\theta}{1+\theta\lambda_1}} \hat{P}_i^{-\frac{\theta}{1+\theta\lambda_1}} \hat{\Pi}_j^{-\frac{\theta}{1+\theta\lambda_1}} \\
&+ \sum_j \left(\frac{\Xi^2_{ij}}{\delta_i + \sum_j \Xi^1_{ij} + \sum_j \Xi^2_{ij}}\right) \hat{\bar{s}}_{ii'}^{-\frac{\theta}{1+\theta\lambda_2}} \hat{\tau}_{i'j'}^{-\theta} \hat{\bar{s}}_{j'j}^{-\frac{\theta}{1+\theta\lambda_2}} \\
&\times \hat{P}_i^{-\frac{\theta}{1+\theta\lambda_2}} \hat{\Pi}_j^{-\frac{\theta}{1+\theta\lambda_2}} \left(\sum_l \frac{\Xi^2_{i'l'}}{\sum_{l'} \Xi^2_{i'l'}} \hat{\tau}_{i'l'}^{-\theta} \hat{s}_{l'l}^{-\theta} \hat{\Pi}_l^{-\theta}\right)^{-\frac{\theta\lambda_2}{1+\theta\lambda_2}} \left(\sum_l \frac{\Xi^2_{j'l'}}{\sum_{l'} \Xi^2_{j'l'}} \hat{\tau}_{j'l'}^{-\theta} \hat{s}_{l'l}^{-\theta} \hat{P}_l^{-\theta}\right)^{-\frac{\theta\lambda_2}{1+\theta\lambda_2}}
\end{aligned}$$

$$\begin{aligned}
\hat{P}_i^{-\theta} \hat{\Pi}_i^{-\theta} = &\left(\frac{\gamma_i}{\gamma_i + \sum_j \Xi^1_{ji} + \sum_j \Xi^2_{ji}}\right) \hat{\gamma}_i + \sum_j \left(\frac{\Xi^1_{ji}}{\gamma_i + \sum_j \Xi^1_{ji} + \sum_j \Xi^2_{ji}}\right) \hat{\bar{t}}_{ji}^{-\frac{\theta}{1+\theta\lambda_1}} \hat{\Pi}_i^{-\frac{\theta}{1+\theta\lambda_1}} \hat{P}_j^{-\frac{\theta}{1+\theta\lambda_1}} \\
&+ \sum_j \left(\frac{\Xi^2_{ji}}{\gamma_i + \sum_j \Xi^1_{ji} + \sum_j \Xi^2_{ji}}\right) \hat{\bar{s}}_{jj'}^{-\frac{\theta}{1+\theta\lambda_2}} \hat{\tau}_{j'i'}^{-\theta} \hat{\bar{s}}_{i'i}^{-\frac{\theta}{1+\theta\lambda_2}} \\
&\times \hat{\Pi}_i^{-\frac{\theta}{1+\theta\lambda_2}} \hat{P}_j^{-\frac{\theta}{1+\theta\lambda_2}} \left(\sum_l \frac{\Xi^2_{i'l'}}{\sum_{l'} \Xi^2_{i'l'}} \hat{\tau}_{i'l'}^{-\theta} \hat{s}_{l'l}^{-\theta} \hat{P}_l^{-\theta}\right)^{-\frac{\theta\lambda_2}{1+\theta\lambda_2}} \left(\sum_l \frac{\Xi^2_{j'l'}}{\sum_{l'} \Xi^2_{j'l'}} \hat{\tau}_{j'l'}^{-\theta} \hat{s}_{l'l}^{-\theta} \hat{\Pi}_l^{-\theta}\right)^{-\frac{\theta\lambda_2}{1+\theta\lambda_2}}
\end{aligned}$$

C.1.2 Additional assumptions from the Model

For the economic geography model, we have the following definitions for the fixed effects:

$$\delta_i = E_i$$

$$\gamma_i = Y_i$$

Welfare equalization implies:

$$\begin{aligned}
P_i &= \frac{w_i u_i}{W} \Longleftrightarrow \\
P_i &= Y_i \bar{u}_i L_i^{\beta-1} W^{-1} \Longrightarrow \\
\hat{P}_i &= \hat{y}_i \hat{l}_i^{\beta-1} \hat{W}^{-1}
\end{aligned}$$

and

$$\begin{aligned}
\Pi_i &= A_i L_i Y_i^{-\frac{\theta+1}{\partial}} \Longleftrightarrow \\
\Pi_i &= A_i L_i^{\alpha+1} Y_i^{-\frac{\partial+1}{\theta}} \Longrightarrow \\
\hat{\Pi}_i &= \hat{l}_i^{\alpha+1} \hat{y}_i^{-\frac{\theta+1}{\theta}}
\end{aligned}$$

Also recall that we define,

$$\chi \equiv \left(\frac{\bar{L}^{(\alpha+\beta)}}{\bar{W}}\right)^{\theta}$$

which implies

$$\hat{\chi} = \left(\hat{W}\right)^{-\theta}$$

Substituting into the equilibrium conditions, we get,

$$\begin{aligned}
\hat{\Pi}_i^{-\theta} \hat{P}_i^{-\theta} = &\left(\frac{\delta_i}{\delta_i + \sum_j \Xi_{ij}^1 + \sum_j \Xi_{ij}^2}\right) \hat{\delta}_i \\
&+ \sum_j \left(\frac{\Xi_{ij}^1}{\delta_i + \sum_j \Xi_{ij}^1 + \sum_j \Xi_{ij}^2}\right) \hat{t}_{ij}^{-\frac{\theta}{1+\theta\lambda_1}} \hat{P}_i^{-\frac{\theta}{1+\theta\lambda_1}} \hat{\Pi}_j^{-\frac{\theta}{1+\theta\lambda_1}} \\
&+ \sum_j \left(\frac{\Xi_{ij}^2}{\delta_i + \sum_j \Xi_{ij}^1 + \sum_j \Xi_{ij}^2}\right) \hat{\hat{s}}_{ii'}^{-\frac{\theta}{1+\theta\lambda_2}} \hat{\tau}_{i'j'}^{-\theta} \hat{\hat{s}}_{j'j}^{-\frac{\theta}{1+\theta\lambda_2}} \hat{P}_i^{-\frac{\theta}{1+\theta\lambda_2}} \hat{\Pi}_j^{-\frac{\theta}{1+\theta\lambda_2}} \\
&\times \left(\sum_l \frac{\Xi_{i'l'}^2}{\sum_{l'} \Xi_{i'l'}^2} \hat{\tau}_{i'l'}^{-\theta} \hat{s}_{l'l}^{-\theta} \hat{\Pi}_l^{-\theta}\right)^{-\frac{\theta\lambda_2}{1+\theta\lambda_2}} \left(\sum_l \frac{\Xi_{j'l'}^2}{\sum_{l'} \Xi_{j'l'}^2} \hat{\tau}_{j'l'}^{-\theta} \hat{s}_{l'l}^{-\theta} \hat{P}_l^{-\theta}\right)^{-\frac{\theta\lambda_2}{1+\theta\lambda_2}}
\end{aligned}$$

$$\begin{aligned}
\left(\hat{l}_i^{\alpha+1} \hat{y}_i^{-\frac{\theta+1}{\theta}}\right)^{-\theta} \left(\hat{y}_i \hat{l}_i^{\beta-1} \hat{W}^{-1}\right)^{-\theta} = &\left(\frac{E_i}{E_i + \sum_j \Xi_{ij}^1 + \sum_j \Xi_{ij}^2}\right) \hat{y}_i \\
&+ \sum_j \left(\frac{\Xi_{ij}^1}{E_i + \sum_j \Xi_{ij}^1 + \sum_j \Xi_{ij}^2}\right) \hat{t}_{ij}^{-\frac{\theta}{1+\theta\lambda_1}} \left(\hat{y}_i \hat{l}_i^{\beta-1} \hat{W}^{-1}\right)^{-\frac{\theta}{1+\theta\lambda_1}} \left(\hat{l}_j^{\alpha+1} \hat{y}_j^{-\frac{\theta+1}{\theta}}\right)^{-\frac{\theta}{1+\theta\lambda_1}} \\
&+ \sum_j \left(\frac{\Xi_{ij}^2}{E_i + \sum_j \Xi_{ij}^1 + \sum_j \Xi_{ij}^2}\right) \hat{\hat{s}}_{ii'}^{-\frac{\theta}{1+\theta\lambda_2}} \hat{\tau}_{i'j'}^{-\theta} \hat{\hat{s}}_{j'j}^{-\frac{\theta}{1+\theta\lambda_2}} \left(\hat{y}_i \hat{l}_i^{\beta-1} \hat{W}^{-1}\right)^{-\frac{\theta}{1+\theta\lambda_2}} \left(\hat{l}_j^{\alpha+1} \hat{y}_j^{-\frac{\theta+1}{\theta}}\right)^{-\frac{\theta}{1+\theta\lambda_2}} \\
&\times \left(\sum_l \frac{\Xi_{i'l'}^2}{\sum_{l'} \Xi_{i'l'}^2} \hat{\tau}_{i'l'}^{-\theta} \hat{s}_{l'l}^{-\theta} \left(\hat{l}_l^{\alpha+1} \hat{y}_l^{-\frac{\theta+1}{\theta}}\right)^{-\theta}\right)^{-\frac{\theta\lambda_2}{1+\theta\lambda_2}} \left(\sum_l \frac{\Xi_{j'l'}^2}{\sum_{l'} \Xi_{j'l'}^2} \hat{\tau}_{j'l'}^{-\theta} \hat{s}_{l'l}^{-\theta} \left(\hat{y}_l \hat{l}_l^{\beta-1} \hat{W}^{-1}\right)^{-\theta}\right)^{-\frac{\theta\lambda_2}{1+\theta\lambda_2}}
\end{aligned}$$

$$\left(\hat{l}_i^{\alpha+1}\hat{y}_i^{-\frac{\theta+1}{\theta}}\right)^{-\theta}\left(\hat{y}_i\hat{l}_i^{\beta-1}\right)^{-\theta}=\hat{W}^{-\theta}\left(\frac{E_i}{E_i+\sum_j\Xi_{ij}^1+\sum_j\Xi_{ij}^2}\right)\hat{y}_i$$
$$+\hat{W}^{\frac{\theta}{1+\theta\lambda_1}-\theta}\sum_j\left(\frac{\Xi_{ij}^1}{E_i+\sum_j\Xi_{ij}^1+\sum_j\Xi_{ij}^2}\right)\hat{t}_{ij}^{-\frac{\theta}{1+\theta\lambda_1}}\left(\hat{y}_i\hat{l}_i^{\beta-1}\right)^{-\frac{\theta}{1+\theta\lambda_1}}\left(\hat{l}_j^{\alpha+1}\hat{y}_j^{-\frac{\theta+1}{\theta}}\right)^{-\frac{\theta}{1+\theta\lambda_1}}$$
$$+\hat{W}^{\frac{\theta}{1+\theta\lambda_2}-\theta-\frac{\theta\theta\lambda_2}{1+\theta\lambda_2}}\sum_j\left(\frac{\Xi_{ij}^2}{E_i+\sum_j\Xi_{ij}^1+\sum_j\Xi_{ij}^2}\right)\hat{\hat{s}}_{ii'}^{-\frac{\theta}{1+\theta\lambda_2}}\hat{\tau}_{i'j'}^{-\theta}\hat{\hat{s}}_{j'j}^{-\frac{\theta}{1+\theta\lambda_2}}\left(\hat{y}_i\hat{l}_i^{\beta-1}\right)^{-\frac{\theta}{1+\theta\lambda_2}}\left(\hat{l}_j^{\alpha+1}\hat{y}_j^{-\frac{\theta+1}{\theta}}\right)^{-\frac{\theta}{1+\theta\lambda_2}}$$
$$\times\left(\sum_l\frac{\Xi_{i'l'}^2}{\sum_{l'}\Xi_{i'l'}^2}\hat{\tau}_{i'l'}^{-\theta}\hat{s}_{l'l}^{-\theta}\left(\hat{l}_l^{\alpha+1}\hat{y}_l^{-\frac{\theta+1}{\theta}}\right)^{-\theta}\right)^{-\frac{\theta\lambda_2}{1+\theta\lambda_2}}\left(\sum_l\frac{\Xi_{j'l'}^2}{\sum_{l'}\Xi_{j'l'}^2}\hat{\tau}_{j'l'}^{-\theta}\hat{s}_{l'l}^{-\theta}\left(\hat{y}_l\hat{l}_l^{\beta-1}\right)^{-\theta}\right)^{-\frac{\theta\lambda_2}{1+\theta\lambda_2}}$$

$$\left(\hat{l}_i^{\alpha+1}\hat{y}_i^{-\frac{\theta+1}{\theta}}\right)^{-\theta}\left(\hat{y}_i\hat{l}_i^{\beta-1}\right)^{-\theta}=\hat{\chi}\left(\frac{E_i}{E_i+\sum_j\Xi_{ij}^1+\sum_j\Xi_{ij}^2}\right)\hat{y}_i$$
$$+\hat{\chi}^{\frac{\theta\lambda_1}{1+\theta\lambda_1}}\sum_j\left(\frac{\Xi_{ij}^1}{E_i+\sum_j\Xi_{ij}^1+\sum_j\Xi_{ij}^2}\right)\hat{t}_{ij}^{-\frac{\theta}{1+\theta\lambda_1}}\left(\hat{y}_i\hat{l}_i^{\beta-1}\right)^{-\frac{\theta}{1+\theta\lambda_1}}\left(\hat{l}_j^{\alpha+1}\hat{y}_j^{-\frac{\theta+1}{\theta}}\right)^{-\frac{\theta}{1+\theta\lambda_1}}$$
$$+\hat{\chi}^{\frac{2\theta\lambda_2}{1+\theta\lambda_2}}\sum_j\left(\frac{\Xi_{ij}^2}{E_i+\sum_j\Xi_{ij}^1+\sum_j\Xi_{ij}^2}\right)\hat{\hat{s}}_{ii'}^{-\frac{\theta}{1+\theta\lambda_2}}\hat{\tau}_{i'j'}^{-\theta}\hat{\hat{s}}_{j'j}^{-\frac{\theta}{1+\theta\lambda_2}}\left(\hat{y}_i\hat{l}_i^{\beta-1}\right)^{-\frac{\theta}{1+\theta\lambda_2}}\left(\hat{l}_j^{\alpha+1}\hat{y}_j^{-\frac{\theta+1}{\theta}}\right)^{-\frac{\theta}{1+\theta\lambda_2}}$$
$$\times\left(\sum_l\frac{\Xi_{i'l'}^2}{\sum_{l'}\Xi_{i'l'}^2}\hat{\tau}_{i'l'}^{-\theta}\hat{s}_{l'l}^{-\theta}\left(\hat{l}_l^{\alpha+1}\hat{y}_l^{-\frac{\theta+1}{\theta}}\right)^{-\theta}\right)^{-\frac{\theta\lambda_2}{1+\theta\lambda_2}}\left(\sum_l\frac{\Xi_{j'l'}^2}{\sum_{l'}\Xi_{j'l'}^2}\hat{\tau}_{j'l'}^{-\theta}\hat{s}_{l'l}^{-\theta}\left(\hat{y}_l\hat{l}_l^{\beta-1}\right)^{-\theta}\right)^{-\frac{\theta\lambda_2}{1+\theta\lambda_2}}$$

$$\hat{l}_i^{-\theta(\alpha+\beta)}\hat{y}_i=\hat{\chi}\left(\frac{E_i}{E_i+\sum_j\Xi_{ij}^1+\sum_j\Xi_{ij}^2}\right)\hat{y}_i$$
$$+\hat{\chi}^{\frac{\theta\lambda_1}{1+\theta\lambda_1}}\left(\hat{y}_i\hat{l}_i^{\beta-1}\right)^{-\frac{\theta}{1+\theta\lambda_1}}\sum_j\left(\frac{\Xi_{ij}^1}{E_i+\sum_j\Xi_{ij}^1+\sum_j\Xi_{ij}^2}\right)\hat{t}_{ij}^{-\frac{\theta}{1+\theta\lambda_1}}\hat{l}_j^{\frac{-\theta(\alpha+1)}{1+\theta\lambda_1}}\hat{y}_j^{\frac{\theta+1}{1+\theta\lambda_1}}$$
$$+\hat{\chi}^{\frac{2\theta\lambda_2}{1+\theta\lambda_2}}\left(\hat{y}_i\hat{l}_i^{\beta-1}\right)^{-\frac{\theta}{1+\theta\lambda_2}}\sum_j\left(\frac{\Xi_{ij}^2}{E_i+\sum_j\Xi_{ij}^1+\sum_j\Xi_{ij}^2}\right)\hat{\hat{s}}_{ii'}^{-\frac{\theta}{1+\theta\lambda_2}}\hat{\tau}_{i'j'}^{-\theta}\hat{\hat{s}}_{j'j}^{-\frac{\theta}{1+\theta\lambda_2}}\hat{l}_j^{\frac{-\theta(\alpha+1)}{1+\theta\lambda_2}}\hat{y}_j^{\frac{\theta+1}{1+\theta\lambda_2}}$$
$$\times\left(\sum_l\frac{\Xi_{i'l'}^2}{\sum_{l'}\Xi_{i'l'}^2}\hat{\tau}_{i'l'}^{-\theta}\hat{s}_{l'l}^{-\theta}\left(\hat{l}_l^{\alpha+1}\hat{y}_l^{-\frac{\theta+1}{\theta}}\right)^{-\theta}\right)^{-\frac{\theta\lambda_2}{1+\theta\lambda_2}}\left(\sum_l\frac{\Xi_{j'l'}^2}{\sum_{l'}\Xi_{j'l'}^2}\hat{\tau}_{j'l'}^{-\theta}\hat{s}_{l'l}^{-\theta}\left(\hat{y}_l\hat{l}_l^{\beta-1}\right)^{-\theta}\right)^{-\frac{\theta\lambda_2}{1+\theta\lambda_2}}$$

$$\hat{l}_i^{\frac{-\theta(1+\alpha+\theta\lambda_1(\alpha+\beta))}{1+\theta\lambda_1}}\hat{y}_i^{\frac{1+\theta\lambda_1+\theta}{1+\theta\lambda_1}}=\hat{\chi}\left(\frac{E_i}{E_i+\sum_j\Xi_{ij}^1+\sum_j\Xi_{ij}^2}\right)\hat{y}_i^{\frac{1+\theta\lambda_1+\theta}{1+\theta\lambda_1}}\hat{l}_i^{\frac{\theta(\beta-1)}{1+\theta\lambda_1}}$$
$$+\hat{\chi}^{\frac{\theta\lambda_1}{1+\theta\lambda_1}}\sum_j\left(\frac{\Xi_{ij}^1}{E_i+\sum_j\Xi_{ij}^1+\sum_j\Xi_{ij}^2}\right)\hat{t}_{ij}^{-\frac{\theta}{1+\theta\lambda_1}}\hat{l}_j^{\frac{-\theta(\alpha+1)}{1+\theta\lambda_1}}\hat{y}_j^{\frac{\theta+1}{1+\theta\lambda_1}}$$
$$+\hat{\chi}^{\frac{2\theta\lambda_2}{1+\theta\lambda_2}}\left(\hat{y}_i\hat{l}_i^{\beta-1}\right)^{-\frac{\theta}{1+\theta\lambda_2}+\frac{\theta}{1+\theta\lambda_1}}\sum_j\left(\frac{\Xi_{ij}^2}{E_i+\sum_j\Xi_{ij}^1+\sum_j\Xi_{ij}^2}\right)\hat{\hat{s}}_{ii'}^{-\frac{\theta}{1+\theta\lambda_2}}\hat{\tau}_{i'j'}^{-\theta}\hat{\hat{s}}_{j'j}^{-\frac{\theta}{1+\theta\lambda_2}}\hat{l}_j^{\frac{-\theta(\alpha+1)}{1+\theta\lambda_2}}\hat{y}_j^{\frac{\theta+1}{1+\theta\lambda_2}}$$
$$\times\left(\sum_l\frac{\Xi_{i'l'}^2}{\sum_{l'}\Xi_{i'l'}^2}\hat{\tau}_{i'l'}^{-\theta}\hat{s}_{l'l}^{-\theta}\left(\hat{l}_l^{\alpha+1}\hat{y}_l^{-\frac{\theta+1}{\theta}}\right)^{-\theta}\right)^{-\frac{\theta\lambda_2}{1+\theta\lambda_2}}\left(\sum_l\frac{\Xi_{j'l'}^2}{\sum_{l'}\Xi_{j'l'}^2}\hat{\tau}_{j'l'}^{-\theta}\hat{s}_{l'l}^{-\theta}\left(\hat{y}_l\hat{l}_l^{\beta-1}\right)^{-\theta}\right)^{-\frac{\theta\lambda_2}{1+\theta\lambda_2}}$$

$$
\begin{aligned}
\hat{l}_i^{\frac{-\theta(1+\alpha+\theta\lambda_1(\alpha+\beta))}{1+\theta\lambda_1}} \hat{y}_i^{\frac{1+\theta\lambda_1+\theta}{1+\theta\lambda_1}} = {} & \hat{\chi}\left(\frac{E_i}{E_i+\sum_j \Xi_{ij}^1+\sum_j \Xi_{ij}^2}\right)\hat{y}_i^{\frac{1+\theta\lambda_1+\theta}{1+\theta\lambda_1}} \hat{l}_i^{\frac{\theta(\beta-1)}{1+\theta\lambda_1}} \\
& + \hat{\chi}^{\frac{\theta\lambda_1}{1+\theta\lambda_1}} \sum_j \left(\frac{\Xi_{ij}^1}{E_i+\sum_j \Xi_{ij}^1+\sum_j \Xi_{ij}^2}\right) \hat{t}_{ij}^{-\frac{\theta}{1+\theta\lambda_1}} \hat{l}_j^{\frac{-\theta(\alpha+1)}{1+\theta\lambda_1}} \hat{y}_j^{\frac{\theta+1}{1+\theta\lambda_1}} \\
& + \hat{\chi}^{\frac{2\theta\lambda_2}{1+\theta\lambda_2}} \left(\hat{y}_i \hat{l}_i^{\beta-1}\right)^{\frac{\theta^2(\lambda_1-\lambda_2)}{(1+\theta\lambda_1)(1+\theta\lambda_2)}} \sum_j \left(\frac{\Xi_{ij}^2}{E_i+\sum_j \Xi_{ij}^1+\sum_j \Xi_{ij}^2}\right) \hat{\hat{s}}_{ii'}^{-\frac{\theta}{1+\theta\lambda_2}} \hat{\tau}_{i'j'}^{-\theta} \hat{\hat{s}}_{j'j}^{-\frac{\theta}{1+\theta\lambda_2}} \hat{l}_j^{\frac{-\theta(\alpha+1)}{1+\theta\lambda_2}} \hat{y}_j^{\frac{\theta+1}{1+\theta\lambda_2}} \\
& \times \left(\sum_l \frac{\Xi_{i'l'}^2}{\sum_{l'} \Xi_{i'l'}^2} \hat{\tau}_{i'l'}^{-\theta} \hat{s}_{l'l}^{-\theta} \left(\hat{l}_l^{\alpha+1} \hat{y}_l^{-\frac{\theta+1}{\theta}}\right)^{-\theta}\right)^{-\frac{\theta\lambda_2}{1+\theta\lambda_2}} \left(\sum_l \frac{\Xi_{j'l'}^2}{\sum_{l'} \Xi_{j'l'}^2} \hat{\tau}_{j'l'}^{-\theta} \hat{s}_{l'l}^{-\theta} \left(\hat{y}_l \hat{l}_l^{\beta-1}\right)^{-\theta}\right)^{-\frac{\theta\lambda_2}{1+\theta\lambda_2}}
\end{aligned}
$$

and:

$$
\begin{aligned}
\hat{P}_i^{-\theta} \hat{\Pi}_i^{-\theta} = {} & \left(\frac{Y_i}{Y_i+\sum_j \Xi_{ji}^1+\sum_j \Xi_{ji}^2}\right)\hat{y}_i \\
& + \sum_j \left(\frac{\Xi_{ji}^1}{Y_i+\sum_j \Xi_{ji}^1+\sum_j \Xi_{ji}^2}\right) \hat{\hat{t}}_{ji}^{-\frac{\theta}{1+\theta\lambda_1}} \hat{\Pi}_i^{-\frac{\theta}{1+\theta\lambda_1}} \hat{P}_j^{-\frac{\theta}{1+\theta\lambda_1}} \\
& + \sum_j \left(\frac{\Xi_{ji}^2}{Y_i+\sum_j \Xi_{ji}^1+\sum_j \Xi_{ji}^2}\right) \hat{\hat{s}}_{jj'}^{-\frac{\theta}{1+\theta\lambda_2}} \hat{\tau}_{j'i'}^{-\theta} \hat{\hat{s}}_{i'i}^{-\frac{\theta}{1+\theta\lambda_2}} \hat{\Pi}_i^{-\frac{\theta}{1+\theta\lambda_2}} \hat{P}_j^{-\frac{\theta}{1+\theta\lambda_2}} \\
& \times \left(\sum_l \frac{\Xi_{i'l'}^2}{\sum_{l'} \Xi_{i'l'}^2} \hat{\tau}_{i'l'}^{-\theta} \hat{s}_{l'l}^{-\theta} \hat{P}_l^{-\theta}\right)^{-\frac{\theta\lambda_2}{1+\theta\lambda_2}} \left(\sum_l \frac{\Xi_{j'l'}^2}{\sum_{l'} \Xi_{j'l'}^2} \hat{\tau}_{j'l'}^{-\theta} \hat{s}_{l'l}^{-\theta} \hat{\Pi}_l^{-\theta}\right)^{-\frac{\theta\lambda_2}{1+\theta\lambda_2}}
\end{aligned}
$$

$$
\begin{aligned}
\left(\hat{l}_i^{\alpha+1} \hat{y}_i^{-\frac{\theta+1}{\theta}}\right)^{-\theta} \left(\hat{y}_i \hat{l}_i^{\beta-1} \hat{W}^{-1}\right)^{-\theta} = {} & \left(\frac{Y_i}{Y_i+\sum_j \Xi_{ji}^1+\sum_j \Xi_{ji}^2}\right)\hat{y}_i \\
& + \sum_j \left(\frac{\Xi_{ij}^1}{Y_i+\sum_j \Xi_{ji}^1+\sum_j \Xi_{ji}^2}\right) \hat{\hat{t}}_{ji}^{-\frac{\theta}{1+\theta\lambda_1}} \left(\hat{y}_j \hat{l}_j^{\beta-1} \hat{W}^{-1}\right)^{-\frac{\theta}{1+\theta\lambda_1}} \left(\hat{l}_i^{\alpha+1} \hat{y}_i^{-\frac{\theta+1}{\theta}}\right)^{-\frac{\theta}{1+\theta\lambda_1}} \\
& + \sum_j \left(\frac{\Xi_{ij}^2}{Y_i+\sum_j \Xi_{ji}^1+\sum_j \Xi_{ji}^2}\right) \hat{\hat{s}}_{jj'}^{-\frac{\theta}{1+\theta\lambda_2}} \hat{\tau}_{j'i'}^{-\theta} \hat{\hat{s}}_{i'i}^{-\frac{\theta}{1+\theta\lambda_2}} \left(\hat{y}_j \hat{l}_j^{\beta-1} \hat{W}^{-1}\right)^{-\frac{\theta}{1+\theta\lambda_2}} \left(\hat{l}_i^{\alpha+1} \hat{y}_i^{-\frac{\theta+1}{\theta}}\right)^{-\frac{\theta}{1+\theta\lambda_2}} \\
& \times \left(\sum_l \frac{\Xi_{i'l'}^2}{\sum_{l'} \Xi_{i'l'}^2} \hat{\tau}_{i'l'}^{-\theta} \hat{s}_{l'l}^{-\theta} \left(\hat{y}_l \hat{l}_l^{\beta-1} \hat{W}^{-1}\right)^{-\theta}\right)^{-\frac{\theta\lambda_2}{1+\theta\lambda_2}} \left(\sum_l \frac{\Xi_{j'l'}^2}{\sum_{l'} \Xi_{j'l'}^2} \hat{\tau}_{j'l'}^{-\theta} \hat{s}_{l'l}^{-\theta} \left(\hat{l}_l^{\alpha+1} \hat{y}_l^{-\frac{\theta+1}{\theta}}\right)^{-\theta}\right)^{-\frac{\theta\lambda_2}{1+\theta\lambda_2}}
\end{aligned}
$$

$$
\begin{aligned}
\left(\hat{l}_i^{\alpha+1} \hat{y}_i^{-\frac{\theta+1}{\theta}}\right)^{-\theta} \left(\hat{y}_i \hat{l}_i^{\beta-1}\right)^{-\theta} = {} & \hat{W}^{-\theta} \left(\frac{Y_i}{Y_i+\sum_j \Xi_{ji}^1+\sum_j \Xi_{ji}^2}\right)\hat{y}_i \\
& + \hat{W}^{\frac{\theta}{1+\theta\lambda_1}-\theta} \sum_j \left(\frac{\Xi_{ij}^1}{Y_i+\sum_j \Xi_{ji}^1+\sum_j \Xi_{ji}^2}\right) \hat{\hat{t}}_{ji}^{-\frac{\theta}{1+\theta\lambda_1}} \left(\hat{y}_j \hat{l}_j^{\beta-1}\right)^{-\frac{\theta}{1+\theta\lambda_1}} \left(\hat{l}_i^{\alpha+1} \hat{y}_i^{-\frac{\theta+1}{\theta}}\right)^{-\frac{\theta}{1+\theta\lambda_1}} \\
& + \hat{W}^{\frac{\theta}{1+\theta\lambda_2}-\theta-\frac{\theta\theta\lambda_2}{1+\theta\lambda_2}} \sum_j \left(\frac{\Xi_{ij}^2}{Y_i+\sum_j \Xi_{ji}^1+\sum_j \Xi_{ji}^2}\right) \hat{\hat{s}}_{jj'}^{-\frac{\theta}{1+\theta\lambda_2}} \hat{\tau}_{j'i'}^{-\theta} \hat{\hat{s}}_{i'i}^{-\frac{\theta}{1+\theta\lambda_2}} \left(\hat{y}_j \hat{l}_j^{\beta-1}\right)^{-\frac{\theta}{1+\theta\lambda_2}} \left(\hat{l}_i^{\alpha+1} \hat{y}_i^{-\frac{\theta+1}{\theta}}\right)^{-\frac{\theta}{1+\theta\lambda_2}} \\
& \times \left(\sum_l \frac{\Xi_{i'l'}^2}{\sum_{l'} \Xi_{i'l'}^2} \hat{\tau}_{i'l'}^{-\theta} \hat{s}_{l'l}^{-\theta} \left(\hat{y}_l \hat{l}_l^{\beta-1}\right)^{-\theta}\right)^{-\frac{\theta\lambda_2}{1+\theta\lambda_2}} \left(\sum_l \frac{\Xi_{j'l'}^2}{\sum_{l'} \Xi_{j'l'}^2} \hat{\tau}_{j'l'}^{-\theta} \hat{s}_{l'l}^{-\theta} \left(\hat{l}_l^{\alpha+1} \hat{y}_l^{-\frac{\theta+1}{\theta}}\right)^{-\theta}\right)^{-\frac{\theta\lambda_2}{1+\theta\lambda_2}}
\end{aligned}
$$

$$
\begin{aligned}
\left(\hat{l}_i^{\alpha+1}\hat{y}_i^{-\frac{\theta+1}{\theta}}\right)^{-\theta}\left(\hat{y}_i\hat{l}_i^{\beta-1}\right)^{-\theta} &= \hat{\chi}\left(\frac{Y_i}{Y_i+\sum_j \Xi_{ji}^1+\sum_j \Xi_{ji}^2}\right)\hat{y}_i \\
&+\hat{\chi}^{\frac{\theta\lambda_1}{1+\theta\lambda_1}}\sum_j\left(\frac{\Xi_{ij}^1}{Y_i+\sum_j \Xi_{ji}^1+\sum_j \Xi_{ji}^2}\right)\hat{\bar{t}}_{ji}^{-\frac{\theta}{1+\theta\lambda_1}}\left(\hat{y}_j\hat{l}_j^{\beta-1}\right)^{-\frac{\theta}{1+\theta\lambda_1}}\left(\hat{l}_i^{\alpha+1}\hat{y}_i^{-\frac{\theta+1}{\theta}}\right)^{-\frac{\theta}{1+\theta\lambda_1}} \\
&+\hat{\chi}^{\frac{2\theta\lambda_2}{1+\theta\lambda_2}}\sum_j\left(\frac{\Xi_{ij}^2}{Y_i+\sum_j \Xi_{ji}^1+\sum_j \Xi_{ji}^2}\right)\hat{\bar{s}}_{jj'}^{-\frac{\theta}{1+\theta\lambda_2}}\hat{\tau}_{j'i'}^{-\theta}\hat{\bar{s}}_{i'i}^{-\frac{\theta}{1+\theta\lambda_2}}\left(\hat{y}_j\hat{l}_j^{\beta-1}\right)^{-\frac{\theta}{1+\theta\lambda_2}}\left(\hat{l}_i^{\alpha+1}\hat{y}_i^{-\frac{\theta+1}{\theta}}\right)^{-\frac{\theta}{1+\theta\lambda_2}} \\
&\times\left(\sum_l\frac{\Xi_{i'l'}^2}{\sum_{l'}\Xi_{i'l'}^2}\hat{\tau}_{i'l'}^{-\theta}\hat{s}_{l'l}^{-\theta}\left(\hat{y}_l\hat{l}_l^{\beta-1}\right)^{-\theta}\right)^{-\frac{\theta\lambda_2}{1+\theta\lambda_2}}\left(\sum_l\frac{\Xi_{j'l'}^2}{\sum_{l'}\Xi_{j'l'}^2}\hat{\tau}_{j'l'}^{-\theta}\hat{s}_{l'l}^{-\theta}\left(\hat{l}_l^{\alpha+1}\hat{y}_l^{-\frac{\theta+1}{\theta}}\right)^{-\theta}\right)^{-\frac{\theta\lambda_2}{1+\theta\lambda_2}}
\end{aligned}
$$

$$
\begin{aligned}
\hat{l}_i^{-\theta(\alpha+\beta)}\hat{y}_i &= \hat{\chi}\left(\frac{Y_i}{Y_i+\sum_j \Xi_{ji}^1+\sum_j \Xi_{ji}^2}\right)\hat{y}_i \\
&+\hat{\chi}^{\frac{\theta\lambda_1}{1+\theta\lambda_1}}\left(\hat{l}_i^{\alpha+1}\hat{y}_i^{-\frac{\theta+1}{\theta}}\right)^{-\frac{\theta}{1+\theta\lambda_1}}\sum_j\left(\frac{\Xi_{ij}^1}{Y_i+\sum_j \Xi_{ji}^1+\sum_j \Xi_{ji}^2}\right)\hat{\bar{t}}_{ji}^{-\frac{\theta}{1+\theta\lambda_1}}\hat{l}_j^{\frac{\theta(1-\beta)}{1+\theta\lambda_1}}\hat{y}_j^{-\frac{\theta}{1+\theta\lambda_1}} \\
&+\hat{\chi}^{\frac{2\theta\lambda_2}{1+\theta\lambda_2}}\left(\hat{l}_i^{\alpha+1}\hat{y}_i^{-\frac{\theta+1}{\theta}}\right)^{-\frac{\theta}{1+\theta\lambda_2}}\sum_j\left(\frac{\Xi_{ij}^2}{Y_i+\sum_j \Xi_{ji}^1+\sum_j \Xi_{ji}^2}\right)\hat{\bar{s}}_{jj'}^{-\frac{\theta}{1+\theta\lambda_2}}\hat{\tau}_{j'i'}^{-\theta}\hat{\bar{s}}_{i'i}^{-\frac{\theta}{1+\theta\lambda_2}}\hat{l}_j^{\frac{\theta(1-\beta)}{1+\theta\lambda_2}}\hat{y}_j^{-\frac{\theta}{1+\theta\lambda_2}} \\
&\times\left(\sum_l\frac{\Xi_{i'l'}^2}{\sum_{l'}\Xi_{i'l'}^2}\hat{\tau}_{i'l'}^{-\theta}\hat{s}_{l'l}^{-\theta}\left(\hat{y}_l\hat{l}_l^{\beta-1}\right)^{-\theta}\right)^{-\frac{\theta\lambda_2}{1+\theta\lambda_2}}\left(\sum_l\frac{\Xi_{j'l'}^2}{\sum_{l'}\Xi_{j'l'}^2}\hat{\tau}_{j'l'}^{-\theta}\hat{s}_{l'l}^{-\theta}\left(\hat{l}_l^{\alpha+1}\hat{y}_l^{-\frac{\theta+1}{\theta}}\right)^{-\theta}\right)^{-\frac{\theta\lambda_2}{1+\theta\lambda_2}}
\end{aligned}
$$

dividing by $\left(\hat{l}_i^{\alpha+1}\hat{y}_i^{-\frac{\theta+1}{\theta}}\right)^{-\frac{\theta}{1+\theta\lambda_1}}$, we obtain,

$$
\begin{aligned}
\hat{l}_i^{\frac{-\theta(1+\alpha+\theta\lambda_1(\alpha+\beta))}{1+\theta\lambda_1}}\hat{y}_i^{\frac{-\theta(1-\lambda_1)}{1+\theta\lambda_1}} &= \hat{\chi}\left(\frac{Y_i}{Y_i+\sum_j \Xi_{ji}^1+\sum_j \Xi_{ji}^2}\right)\hat{y}_i^{\frac{-\theta(1-\lambda_1)}{1+\theta\lambda_1}}\hat{l}_i^{\frac{\theta(\alpha+1)}{1+\theta\lambda_1}} \\
&+\hat{\chi}^{\frac{\theta\lambda_1}{1+\theta\lambda_1}}\sum_j\left(\frac{\Xi_{ij}^1}{Y_i+\sum_j \Xi_{ji}^1+\sum_j \Xi_{ji}^2}\right)\hat{\bar{t}}_{ji}^{-\frac{\theta}{1+\theta\lambda_1}}\hat{l}_j^{\frac{\theta(1-\beta)}{1+\theta\lambda_1}}\hat{y}_j^{-\frac{\theta}{1+\theta\lambda_1}} \\
&+\hat{\chi}^{\frac{2\theta\lambda_2}{1+\theta\lambda_2}}\left(\hat{l}_i^{\alpha+1}\hat{y}_i^{-\frac{\theta+1}{\theta}}\right)^{-\frac{\theta}{1+\theta\lambda_2}+\frac{\theta}{1+\theta\lambda_1}}\sum_j\left(\frac{\Xi_{ij}^2}{Y_i+\sum_j \Xi_{ji}^1+\sum_j \Xi_{ji}^2}\right)\hat{\bar{s}}_{jj'}^{-\frac{\theta}{1+\theta\lambda_2}}\hat{\tau}_{j'i'}^{-\theta}\hat{\bar{s}}_{i'i}^{-\frac{\theta}{1+\theta\lambda_2}}\hat{l}_j^{\frac{\theta(1-\beta)}{1+\theta\lambda_2}}\hat{y}_j^{-\frac{\theta}{1+\theta\lambda_2}} \\
&\times\left(\sum_l\frac{\Xi_{i'l'}^2}{\sum_{l'}\Xi_{i'l'}^2}\hat{\tau}_{i'l'}^{-\theta}\hat{s}_{l'l}^{-\theta}\left(\hat{y}_l\hat{l}_l^{\beta-1}\right)^{-\theta}\right)^{-\frac{\theta\lambda_2}{1+\theta\lambda_2}}\left(\sum_l\frac{\Xi_{j'l'}^2}{\sum_{l'}\Xi_{j'l'}^2}\hat{\tau}_{j'l'}^{-\theta}\hat{s}_{l'l}^{-\theta}\left(\hat{l}_l^{\alpha+1}\hat{y}_l^{-\frac{\theta+1}{\theta}}\right)^{-\theta}\right)^{-\frac{\theta\lambda_2}{1+\theta\lambda_2}}
\end{aligned}
$$

$$
\begin{aligned}
\hat{l}_i^{\frac{-\theta(1+\alpha+\theta\lambda_1(\alpha+\beta))}{1+\theta\lambda_1}}\hat{y}_i^{\frac{-\theta(1-\lambda_1)}{1+\theta\lambda_1}} &= \hat{\chi}\left(\frac{Y_i}{Y_i+\sum_j \Xi_{ji}^1+\sum_j \Xi_{ji}^2}\right)\hat{y}_i^{\frac{-\theta(1-\lambda_1)}{1+\theta\lambda_1}}\hat{l}_i^{\frac{\theta(\alpha+1)}{1+\theta\lambda_1}} \\
&+\hat{\chi}^{\frac{\theta\lambda_1}{1+\theta\lambda_1}}\sum_j\left(\frac{\Xi_{ij}^1}{Y_i+\sum_j \Xi_{ji}^1+\sum_j \Xi_{ji}^2}\right)\hat{\bar{t}}_{ji}^{-\frac{\theta}{1+\theta\lambda_1}}\hat{l}_j^{\frac{\theta(1-\beta)}{1+\theta\lambda_1}}\hat{y}_j^{-\frac{\theta}{1+\theta\lambda_1}} \\
&+\hat{\chi}^{\frac{2\theta\lambda_2}{1+\theta\lambda_2}}\left(\hat{l}_i^{\alpha+1}\hat{y}_i^{-\frac{\theta+1}{\theta}}\right)^{\frac{\theta^2(\lambda_1-\lambda_2)}{(1+\theta\lambda_1)(1+\theta\lambda_2)}}\sum_j\left(\frac{\Xi_{ij}^2}{Y_i+\sum_j \Xi_{ji}^1+\sum_j \Xi_{ji}^2}\right)\hat{\bar{s}}_{jj'}^{-\frac{\theta}{1+\theta\lambda_2}}\hat{\tau}_{j'i'}^{-\theta}\hat{\bar{s}}_{i'i}^{-\frac{\theta}{1+\theta\lambda_2}}\hat{l}_j^{\frac{\theta(1-\beta)}{1+\theta\lambda_2}}\hat{y}_j^{-\frac{\theta}{1+\theta\lambda_2}} \\
&\times\left(\sum_l\frac{\Xi_{i'l'}^2}{\sum_{l'}\Xi_{i'l'}^2}\hat{\tau}_{i'l'}^{-\theta}\hat{s}_{l'l}^{-\theta}\left(\hat{y}_l\hat{l}_l^{\beta-1}\right)^{-\theta}\right)^{-\frac{\theta\lambda_2}{1+\theta\lambda_2}}\left(\sum_l\frac{\Xi_{j'l'}^2}{\sum_{l'}\Xi_{j'l'}^2}\hat{\tau}_{j'l'}^{-\theta}\hat{s}_{l'l}^{-\theta}\left(\hat{l}_l^{\alpha+1}\hat{y}_l^{-\frac{\theta+1}{\theta}}\right)^{-\theta}\right)^{-\frac{\theta\lambda_2}{1+\theta\lambda_2}}
\end{aligned}
$$

To summarize the economic geography model, one can write the equilibrium system of equations (10) and (11) in changes as:

$$\begin{aligned}
\hat{l}_i^{\frac{-\theta(1+\alpha+\theta\lambda_1(\alpha+\beta))}{1+\theta\lambda_1}} \hat{y}_i^{\frac{1+\theta\lambda_1+\theta}{1+\theta\lambda_1}} &= \hat{\chi} \left(\frac{E_i}{E_i + \sum_j \Xi_{ij}^1 + \sum_j \Xi_{ij}^2} \right) \hat{y}_i^{\frac{1+\theta\lambda_1+\theta}{1+\theta\lambda_1}} \hat{l}_i^{\frac{\theta(\beta-1)}{1+\theta\lambda_1}} \\
&+ \hat{\chi}^{\frac{\theta\lambda_1}{1+\theta\lambda_1}} \sum_j \left(\frac{\Xi_{ij}^1}{E_i + \sum_j \Xi_{ij}^1 + \sum_j \Xi_{ij}^2} \right) \hat{t}_{ij}^{-\frac{\theta}{1+\theta\lambda_1}} \hat{l}_j^{\frac{-\theta(\alpha+1)}{1+\theta\lambda_1}} \hat{y}_j^{\frac{\theta+1}{1+\theta\lambda_1}} \\
&+ \hat{\chi}^{\frac{2\theta\lambda_2}{1+\theta\lambda_2}} \left(\hat{y}_i \hat{l}_i^{\beta-1} \right)^{\frac{\theta^2(\lambda_1-\lambda_2)}{(1+\theta\lambda_1)(1+\theta\lambda_2)}} \sum_j \left(\frac{\Xi_{ij}^2}{E_i + \sum_j \Xi_{ij}^1 + \sum_j \Xi_{ij}^2} \right) \hat{\hat{s}}_{ii'}^{-\frac{\theta}{1+\theta\lambda_2}} \hat{\tau}_{i'j'}^{-\theta} \hat{\hat{s}}_{j'j}^{-\frac{\theta}{1+\theta\lambda_2}} \hat{l}_j^{\frac{-\theta(\alpha+1)}{1+\theta\lambda_2}} \hat{y}_j^{\frac{\theta+1}{1+\theta\lambda_2}} \\
&\times \left(\sum_l \frac{\Xi_{i'l'}^2}{\sum_{l'} \Xi_{i'l'}^2} \hat{\tau}_{i'l'}^{-\theta} \hat{s}_{l'l}^{-\theta} \left(\hat{l}_l^{\alpha+1} \hat{y}_l^{-\frac{\theta+1}{\theta}} \right)^{-\theta} \right)^{-\frac{\theta\lambda_2}{1+\theta\lambda_2}} \left(\sum_l \frac{\Xi_{j'l'}^2}{\sum_{l'} \Xi_{j'l'}^2} \hat{\tau}_{j'l'}^{-\theta} \hat{s}_{l'l}^{-\theta} \left(\hat{y}_l \hat{l}_l^{\beta-1} \right)^{-\theta} \right)^{-\frac{\theta\lambda_2}{1+\theta\lambda_2}}
\end{aligned}$$

$$\begin{aligned}
\hat{l}_i^{\frac{-\theta(1+\alpha+\theta\lambda_1(\alpha+\beta))}{1+\theta\lambda_1}} \hat{y}_i^{\frac{-\theta(1-\lambda_1)}{1+\theta\lambda_1}} &= \hat{\chi} \left(\frac{Y_i}{Y_i + \sum_j \Xi_{ji}^1 + \sum_j \Xi_{ji}^2} \right) \hat{y}_i^{\frac{-\theta(1-\lambda_1)}{1+\theta\lambda_1}} \hat{l}_i^{\frac{\theta(\alpha+1)}{1+\theta\lambda_1}} \\
&+ \hat{\chi}^{\frac{\theta\lambda_1}{1+\theta\lambda_1}} \sum_j \left(\frac{\Xi_{ij}^1}{Y_i + \sum_j \Xi_{ji}^1 + \sum_j \Xi_{ji}^2} \right) \hat{\hat{t}}_{ji}^{-\frac{\theta}{1+\theta\lambda_1}} \hat{l}_j^{\frac{\theta(1-\beta)}{1+\theta\lambda_1}} \hat{y}_j^{-\frac{\theta}{1+\theta\lambda_1}} \\
&+ \hat{\chi}^{\frac{2\theta\lambda_2}{1+\theta\lambda_2}} \left(\hat{l}_i^{\alpha+1} \hat{y}_i^{-\frac{\theta+1}{\theta}} \right)^{\frac{\theta^2(\lambda_1-\lambda_2)}{(1+\theta\lambda_1)(1+\theta\lambda_2)}} \sum_j \left(\frac{\Xi_{ij}^2}{Y_i + \sum_j \Xi_{ji}^1 + \sum_j \Xi_{ji}^2} \right) \hat{\hat{s}}_{jj'}^{-\frac{\theta}{1+\theta\lambda_2}} \hat{\tau}_{j'i'}^{-\theta} \hat{\hat{s}}_{i'i}^{-\frac{\theta}{1+\theta\lambda_2}} \hat{l}_j^{\frac{\theta(1-\beta)}{1+\theta\lambda_2}} \hat{y}_j^{-\frac{\theta}{1+\theta\lambda_2}} \\
&\times \left(\sum_l \frac{\Xi_{i'l'}^2}{\sum_{l'} \Xi_{i'l'}^2} \hat{\tau}_{i'l'}^{-\theta} \hat{s}_{l'l}^{-\theta} \left(\hat{y}_l \hat{l}_l^{\beta-1} \right)^{-\theta} \right)^{-\frac{\theta\lambda_2}{1+\theta\lambda_2}} \left(\sum_l \frac{\Xi_{j'l'}^2}{\sum_{l'} \Xi_{j'l'}^2} \hat{\tau}_{j'l'}^{-\theta} \hat{s}_{l'l}^{-\theta} \left(\hat{l}_l^{\alpha+1} \hat{y}_l^{-\frac{\theta+1}{\theta}} \right)^{-\theta} \right)^{-\frac{\theta\lambda_2}{1+\theta\lambda_2}}
\end{aligned}$$

D Extensions

D.1 International Trade and Ports

Let there by a number of locations that constitute nodes on a tertiary transportation network ("Ports"), i.e. $p, u \in \mathbb{P} = \{1, \ldots, N_3\}$. Ports can be accessed via the primary transportation system only. The total level of international imports and exports at each port is fixed and given by $\left\{E_1^M, \ldots, E_{N_3}^M, E_1^X, \ldots, E_{N_3}^X\right\}$. Adjusting the market clearing condition, and distinguishing between ports and domestic locations along the road network,

$$\begin{aligned}
Y_i &= \sum_{j=1}^{N_1} X_{ij} + \sum_{p=1}^{N_3} X_{ip} \Longleftrightarrow \\
Y_i &= \sum_{j=1}^{N} \tau_{ij}^{-\theta} w_i^{-\theta} A_i^{\theta} E_j P_j^{\theta} + \sum_{p=1}^{N^3} \tilde{\tau}_{ip}^{-\theta} w_i^{-\theta} A_i^{\theta} E_p^X \tilde{P}_p^{\theta} \Longleftrightarrow \\
\frac{Y_i}{A_i^{\theta}} w_i^{\theta} &= \sum_{j=1}^{N} \tau_{ij}^{-\theta} E_j P_j^{\theta} + \sum_{p=1}^{N^3} \tilde{\tau}_{ip}^{-\theta} E_p^X \tilde{P}_p^{\theta} \Longleftrightarrow \\
\bar{A}_i^{-\theta} L_i^{1-\alpha\theta} w_i^{\theta+1} &= \sum_{j=1}^{N} \tau_{ij}^{-\theta} E_j P_j^{\theta} + \sum_{p=1}^{N^3} \tau_{ip}^{-\theta} s_p^{-\theta} E_p^X \tilde{P}_p^{\theta}
\end{aligned}$$

where in the last line we have used the fact that transporting to a port implies the usual cost of transporting across the domestic transportation system plus incurring additional switching costs at the port side, i.e. $\tilde{\tau}_{ip}^{-\theta} = \tau_{ip}^{-\theta} s_p^{-\theta}$. Since the price index also incorporates the switching cost all the port site, i.e. $\tilde{P}_p^{\theta} = s_p^{\theta} P_p^{\theta}$, the term cancels out in what follows. With welfare equalization where $W_j = \frac{w_j}{P_j} u_j \Longleftrightarrow P_j = \frac{w_j}{W_j} u_j$ and overall amenity equation $u_j = \bar{u}_j L_j^{\beta}$ this becomes:

$$\begin{aligned}
\bar{A}_i^{-\theta} L_i^{1-\alpha\theta} w_i^{\theta+1} &= \sum_{j=1}^{N} \tau_{ij}^{-\theta} w_j L_j \left(\frac{w_j}{W_j} \bar{u}_j L_j^{\beta}\right)^{\theta} + \sum_{p=1}^{N^3} \tau_{ip}^{-\theta} E_p^X \left(\frac{w_p}{W} \bar{u}_p L_p^{\beta}\right)^{\theta} \Longleftrightarrow \\
\bar{A}_i^{-\theta} L_i^{1-\alpha\theta} w_i^{\theta+1} &= \sum_{j=1}^{N} \tau_{ij}^{-\theta} w_j L_j w_j^{\theta} \bar{u}_j^{\theta} L_j^{\beta\theta} W^{-\theta} + \sum_{p=1}^{N^3} \tau_{ip}^{-\theta} E_p^X w_p^{\theta} \bar{u}_p^{\theta} L_p^{\beta\theta} W^{-\theta} \Longleftrightarrow \\
\bar{A}_i^{-\theta} L_i^{1-\alpha\theta} w_i^{\theta+1} &= \sum_{j=1}^{N} \tau_{ij}^{-\theta} w_j^{\theta+1} \bar{u}_j^{\theta} L_j^{\beta\theta+1} W^{-\theta} + \sum_{p=1}^{N^3} \tau_{ip}^{-\theta} \frac{E_p^X}{y_p} w_p^{\theta+1} \bar{u}_p^{\theta} L_p^{\beta\theta+1} W^{-\theta}.
\end{aligned}$$

Now defining, share of total income in location i $y_i = \frac{Y_i}{Y^W} = \frac{w_i L_i}{Y^W}$ and share of total labor in location i, $l_i = \frac{L_i}{L}$

$$
\begin{aligned}
\bar{A}_i^{-\theta} L_i^{1-\alpha\theta} w_i^{\theta+1} &= \sum_{j=1}^{N} \tau_{ij}^{-\theta} w_j^{\theta+1} \bar{u}_j^{\theta} L_j^{\beta\theta+1} W^{-\theta} + \sum_{p=1}^{N^3} \tau_{ip}^{-\theta} \frac{E_p^X}{y_p} w_p^{\theta+1} \bar{u}_p^{\theta} L_p^{\beta\theta+1} W^{-\theta}. \iff \\
\bar{A}_i^{-\theta} l_i^{1-\alpha\theta} \bar{L}^{1-\alpha\theta} \left(\frac{y_i Y^W}{l_i \bar{L}} \right)^{\theta+1} &= \sum_{j=1}^{N} \tau_{ij}^{-\theta} \left(\frac{y_i Y^W}{l_i \bar{L}} \right)^{\theta+1} \bar{u}_j^{\theta} L_j^{\beta\theta+1} W^{-\theta} + \sum_{p=1}^{N^3} \tau_{ip}^{-\theta} \frac{E_p^X}{y_p} \left(\frac{y_p Y^W}{l_p \bar{L}} \right)^{\theta+1} \bar{u}_p^{\theta} L_p^{\beta\theta+1} W^{-\theta} \iff \\
\bar{A}_i^{-\theta} y_i^{\theta+1} l_i^{-\theta(1+\alpha)} \bar{L}^{\theta(1+\alpha)} \left(Y^W\right)^{\theta+1} &= \left(Y^W\right)^{\theta+1} \bar{L}^{\theta(\beta-1)} W^{-\theta} \sum_{j=1}^{N} \tau_{ij}^{-\theta} \bar{u}_j^{\theta} y_j^{1+\theta} l_j^{\theta(\beta-1)} + \left(Y^W\right)^{\theta+1} \bar{L}^{\theta(\beta-1)} W^{-\theta} \sum_{p=1}^{N^3} \tau_{ip}^{-\theta} \frac{E_p^X}{y_p} y_p^{1+\theta} \bar{u}_p^{\theta} l_p^{\theta} \\
\bar{A}_i^{-\theta} y_i^{1+\theta} l_i^{-\theta(1+\alpha)} &= \frac{\bar{L}^{(\alpha+\beta)\theta}}{W^{\theta}} \sum_{j=1}^{N} \tau_{ij}^{-\theta} \bar{u}_j^{\theta} y_j^{1+\theta} l_j^{\theta(\beta-1)} + \frac{\bar{L}^{(\alpha+\beta)\theta}}{W^{\theta}} \sum_{p=1}^{N^3} \tau_{ip}^{-\theta} \frac{E_p^X}{y_p} y_p^{1+\theta} \bar{u}_p^{\theta} l_p^{\theta(\beta-1)} \\
&= \chi \sum_{j=1}^{N} \tau_{ij}^{-\theta} \bar{u}_j^{\theta} y_j^{1+\theta} l_j^{\theta(\beta-1)} + \chi \sum_{p=1}^{N^3} \tau_{ip}^{-\theta} \frac{E_p^X}{y_p} y_p^{1+\theta} \bar{u}_p^{\theta} l_p^{\theta(\beta-1)}
\end{aligned}
$$

Defining income for port and non-port locations as follows,

$$
\tilde{y}_j^{1+\theta} = \begin{cases} y_j^{1+\theta} & \forall j \neq \mathbb{P} \\ \left(1 + \frac{E_p^X}{y_j}\right) y_j^{1+\theta} & \forall j \in \mathbb{P} \end{cases}
$$

we obtain,

$$
\bar{A}_i^{-\theta} y_i^{1+\theta} l_i^{-\theta(1+\alpha)} = \chi \sum_{j=1}^{N} \tau_{ij}^{-\theta} \bar{u}_j^{\theta} \tilde{y}_j^{1+\theta} l_j^{\theta(\beta-1)} \qquad \forall i
$$

Similarly, we develop the expression for the balanced trade condition,

$$
\begin{aligned}
E_i &= \sum_{j=1}^{N} X_{ji} + \sum_{p=1}^{N_3} X_{pi} \iff \\
E_i &= \sum_{j=1}^{N} \tau_{ji}^{-\theta} w_j^{-\theta} A_j^{\theta} E_i P_i^{\theta} + \sum_{p=1}^{N} \tau_{pi}^{-\theta} \gamma_p^{-\theta} E_i P_i^{\theta} \iff \\
P_i^{-\theta} &= \sum_{j=1}^{N} \tau_{ji}^{-\theta} w_j^{-\theta} \bar{A}_j^{\theta} L_j^{\alpha\theta} + \sum_{p=1}^{N} \tau_{pi}^{-\theta} \gamma_p^{-\theta}
\end{aligned}
$$

Assuming welfare equalization,

$$
W^{\theta} w_i^{-\theta} \bar{u}_i^{-\theta} L_i^{-\beta\theta} = \sum_{j=1}^{N} \tau_{ji}^{-\theta} w_j^{-\theta} \bar{A}_j^{\theta} L_j^{\alpha\theta} + \sum_{p=1}^{N} \tau_{pi}^{-\theta} \left(\frac{\gamma_p}{w_p A_p} \right)^{-\theta} \bar{A}_p^{\theta} L_p^{\alpha\theta} w_p^{-\theta}
$$

Defining income and labor shares as above,

$$W^{\theta}w_i^{-\theta}\bar{u}_i^{-\theta}L_i^{-\beta\theta} = \sum_{j=1}^{N}\tau_{ji}^{-\theta}w_j^{-\theta}\bar{A}_j^{\theta}L_j^{\alpha\theta} + \sum_{p=1}^{N}\tau_{pi}^{-\theta}\left(\frac{\gamma_p}{w_pA_p}\right)^{-\theta}\bar{A}_p^{\theta}L_p^{\alpha\theta}w_p^{-\theta} \iff$$

$$W^{\theta}\left(\frac{y_iY^W}{l_i\bar{L}}\right)^{-\theta}\bar{u}_i^{-\theta}l_i^{-\beta\theta}\bar{L}^{-\beta\theta} = \sum_{j=1}^{N}\tau_{ji}^{-\theta}\left(\frac{y_jY^W}{l_j\bar{L}}\right)^{-\theta}\bar{A}_j^{\theta}l_j^{\alpha\theta}\bar{L}^{\alpha\theta} + \sum_{p=1}^{N}\tau_{pi}^{-\theta}\left(\frac{\gamma_p}{w_pA_p}\right)^{-\theta}\bar{A}_p^{\theta}L_p^{\alpha\theta}\left(\frac{y_jY^W}{l_j\bar{L}}\right)^{-\theta} \iff$$

$$W^{\theta}y_i^{-\theta}l_i^{\theta(1-\beta)}\bar{u}_i^{-\theta}\left(Y^W\right)^{-\theta}\bar{L}^{-\beta\theta} = \left(Y^W\right)^{-\theta}\bar{L}^{\theta(\alpha+1)}\sum_{j=1}^{N}\tau_{ji}^{-\theta}y_j^{-\theta}l_j^{\theta(\alpha+1)}\bar{A}_j^{\theta} + \left(Y^W\right)^{-\theta}\bar{L}^{\theta(\alpha+1)}\sum_{p=1}^{N}\tau_{pi}^{-\theta}\left(\frac{\gamma_p}{w_pA_p}\right)^{-\theta}y_p^{-\theta}l_p^{\theta(\alpha+1)}\bar{A}_p^{\theta} \iff$$

$$\bar{u}_i^{-\theta}y_i^{-\theta}l_i^{\theta(1-\beta)} = \frac{\bar{L}^{(\alpha+\beta)\theta}}{W^{\theta}}\sum_{j=1}^{N}\tau_{ji}^{-\theta}\bar{A}_j^{\theta}y_j^{-\theta}l_j^{\theta(\alpha+1)} + \frac{\bar{L}^{(\alpha+\beta)\theta}}{W^{\theta}}\sum_{p=1}^{N}\tau_{pi}^{-\theta}\left(\frac{\gamma_p}{w_pA_p}\right)^{-\theta}y_p^{-\theta}l_p^{\theta(\alpha+1)}\bar{A}_p^{\theta}$$

$$\bar{u}_i^{-\theta}y_i^{-\theta}l_i^{\theta(1-\beta)} = \frac{\bar{L}^{(\alpha+\beta)\theta}}{W^{\theta}}\sum_{j=1}^{N}\tau_{ji}^{-\theta}\bar{A}_j^{\theta}y_j^{-\theta}l_j^{\theta(\alpha+1)} + \frac{\bar{L}^{(\alpha+\beta)\theta}}{W^{\theta}}\sum_{p=1}^{N}\tau_{pi}^{-\theta}\left(\frac{E_p^M}{Y_p}\right)y_p^{-\theta}l_p^{\theta(\alpha+1)}\bar{A}_p^{\theta}$$

We obtain,

$$\bar{u}_i^{-\theta}y_i^{-\theta}l_i^{\theta(1-\beta)} = \chi\sum_{j=1}^{N}\tau_{ji}^{-\theta}\bar{A}_j^{\theta}y_j^{-\theta}l_j^{\theta(\alpha+1)} + \chi\sum_{p=1}^{N}\tau_{pi}^{-\theta}\frac{E_p^M}{Y_p}y_p^{-\theta}l_p^{\theta(\alpha+1)}\bar{A}_p^{\theta}$$

Defining income for port and non-port locations as follows,

$$\tilde{y}_j^{-\theta} = \begin{cases} y_j^{-\theta} & \forall j \neq \mathbb{P} \\ \left(1 + \frac{E_p^X}{y_j}\right)y_j^{-\theta} & \forall j \in \mathbb{P} \end{cases}$$

we obtain,

$$\bar{u}_i^{-\theta}y_i^{-\theta}l_i^{\theta(1-\beta)} = \chi\sum_{j=1}^{N}\tau_{ji}^{-\theta}\bar{A}_j^{\theta}\tilde{y}_j^{-\theta}l_j^{\theta(\alpha+1)} \qquad \forall i$$

Following the same steps as above, the same counterfactual equilibrium condition holds, but simply replacing the income and expenditure terms appropriately.